Investment Brief Note

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1 金融市场 Common Sense

- **金融机构**：Investment Banking, Commercial Banking, Pension Fund, Mutual Fund, Hedge Fund, **P**rivate **E**quity（股权 / 证券）, **V**enture **C**apital
- **2008金融危机**：**M**ortgage-**B**acked **S**ecurities, **C**ollateralized **D**ebt **O**bligations, **C**redit **D**efault **S**wap
- **Asset Classes**：Fixed Income (Money / Capital Market) + Equity + Derivatives
  - **T**reasury bills, **C**ertificates of **D**eposits (**L**ondon **I**nter**B**ank **O**ffered **R**ate), Commercial Paper, Bankers' Acceptances, Eurodollars, Repo and Reverse Repo, Federal Funds, **E**xchange-**T**raded **F**unds (Asset Management + **A**uthorized **P**articipants)
  - (**I**nflation-**P**rotected) **T**reasury Notes and Bonds (par=1000, semi-annual compounding), Federal Agency Debt, International Bonds, Municipal Bonds, Corporate Bonds, **M**ortgage-**B**acked **S**ecurities
  - Equity, Options (Call/Put), Futures
- **Market**：Primary Market + Secondary Market
  - Private Placement, **I**nitial **P**ublic **O**ffering (underwriters + **S**ecurities and **E**xchange **C**ommission + **C**hina **S**ecurities **R**egulatory **C**ommission / 中国证监会, prospectus + road shows + bookbuilding), **S**easoned **E**quity **O**ffering
  - Limit Order Book: Limit-Buy, Limit-Sell, Stop-Buy, Stop-Loss
    Electronic **C**ommunication **N**etworks, NASDAQ, **N**ew **Y**ork **S**tock **E**xchange (+ Arca), Sarbanes-Oxley Act
    Broker / Dealer - Specialist (market maker)
  - **Margin** $= \frac{Equity}{Value}$, Margin Account, Maintenance Margin, Margin Call

$Assets$		$Liabilities\ \&\ Equity$	
$Shares$	$\$4,000\downarrow$	**Margin Loan**	$\$2,000$
Margin Deposit	$\$0$	$Account\ Equity$	$\$2,000\downarrow$

$Assets$		$Liabilities\ \&\ Equity$	
$Sale$	$\$3,000$	$Short\ Position$	$\$3,000\uparrow$
Margin Deposit	$\$1,500$	$Account\ Equity$	$\$1,500\downarrow$

2 Capital Allocation

It is about speculation, not gamble...

> 1. **Fisher Effect**: $i = r + E(\pi)$, actually: $1 + r = \frac{1+i}{1+\pi}$
> 2. $Expected\ Return\ E(r) = \sum_i p_i r_i$, **or**: $\hat{E(r)} = \frac{1}{n}\sum_{i=1}^{n} r_i$

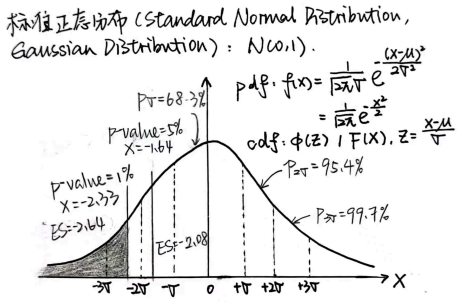
$Variance\ \sigma^2 = \sum_i p_i[r_i - E(r)]^2$, **or**: $\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \overline{r})$

$Estimated\ Skewness = [(\frac{r-\overline{r}}{\hat{\sigma}})]^3$，正/左偏 = 高估risk，负/右偏 = 低估risk

$Estimated\ Kurtosis = [(\frac{r-\overline{r}}{\hat{\sigma}})]^4 - 3$，正峰度 = 低估risk

3. **V**alue **a**t **R**isk：最差的情况损失多少，$VaR(1\%, normal) = Mean - 2.33SD$

   Expected **S**hortfall / CVaR：最差的情况**平均**损失多少



4. Normal Distribution: $E(Geometric\ Avergage) = E(Arithmetic\ Average) - \frac{1}{2}\sigma^2$

- **E**ffective **A**nnualized **R**ates of Return, **A**nnual **P**ercentage **R**eturn
  Holding **P**eriod **R**eturn (= Capital Gain + Dividend Yield)
- **Sharp Ratio** $= \frac{Risk\ Premium}{SD\ of\ excess\ return}$, where $Risk\ Premium = E(r) - r_f$

  Sortino Ratio $= \frac{Risk\ Premium}{LPSD}$, **L**ower **P**artial **S**tandard **D**eviation: count only negative deviations

  Utility Function $= E(r) - \frac{1}{2}A\sigma^2$, (Mean-Variance Utility), is also denoted as **C**ertainty **E**quivalent **R**ate, r and σ in decimal

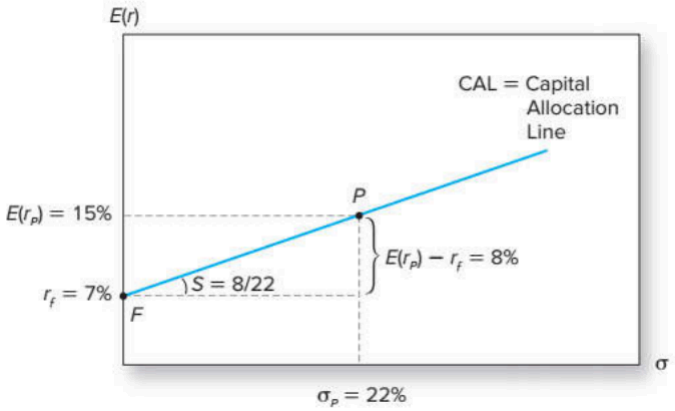
2.1 Markowitz

1. One Risky + Risk-Free

- **C**apital **A**llocation **L**ine (Expected Return - SD) + Utility Function => Best Allocation

$$\begin{cases} E(\mathbf{r_p}) &= r_f + y[E(r_1) - r_f] \\ \sigma_\mathbf{p} &= y\sigma_1 \end{cases}$$

$$\rightarrow E(\mathbf{r_p}) = \frac{E(r_1) - r_f}{\sigma_1} \cdot \sigma_\mathbf{p} + r_f \qquad (CAL)$$



$$Max_{\mathbf{y}}\, U \;=\; E(\mathbf{r_p}) \;-\; \frac{1}{2} A {\sigma_{\mathbf{p}}}^2$$

$$\rightarrow \qquad \mathbf{y^*} \;=\; \frac{E(r_1) - r_f}{A\sigma_1^2} \qquad (best\ allocation)$$

- **C**apital **M**arket **L**ine: CAL for market index + Risk-Free asset (short-term T-bill / money market fund)

2. Two Risky + Risk-Free

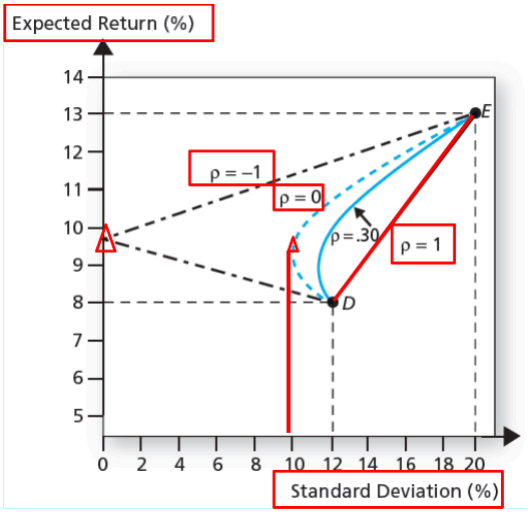
- **Two Risky Assets**

  Efficient Frontier => Optimal Risky Portfolio => Capital Allocation

$$\begin{cases} E(\mathbf{r_{risky}}) \;=\; w_1 E(r_1) + w_2 E(r_2) \\ \sigma^{\mathbf{2}}_{\mathbf{risky}} \;=\; w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2 \cdot Cov(r_1, r_2), \\ \qquad\qquad , \; where \;\; Cov(r_1, r_2) = \rho_{12}\sigma_1\sigma_2 \end{cases}$$

$$(Efficient\ Frontier)$$

$$perfect\ hedge: \;\; \rho_{12} = -1, \;\; \mathbf{w_1} = \frac{\sigma_2}{\sigma_1 + \sigma_2}, \;\; \mathbf{w_2} = \frac{\sigma_1}{\sigma_1 + \sigma_2} \rightarrow \; \sigma_{\mathbf{risky}} = 0$$

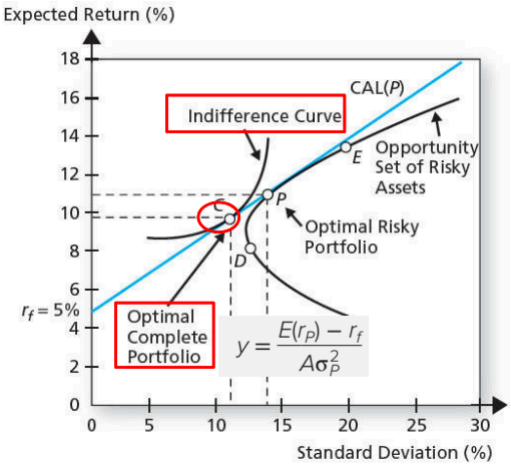


$$\mathbf{(FOC)} \qquad Max_{\mathbf{w_i}}\, S_{risky} \;=\; \frac{E(r_{risky}) - r_f}{\sigma_{risky}} \qquad s.t. \;\; w_1 + w_2 = 1$$

$$solution: w_1 \;=\; \frac{E(R_1)\sigma_2^2 - E(R_2)Cov(R_1, R_2)}{E(R_1)\sigma_2^2 + E(R_2)\sigma_1^2 - [E(R_1) + E(R_2)]Cov(R_1, R_2)}$$

$$w_2 \;=\; 1 \;-\; w_1$$

$$\rightarrow \; E(\mathbf{r_p}) = \frac{E(r^*_{risky}) - r_f}{\sigma^*_{risky}} \cdot \sigma_{\mathbf{p}} + r_f \qquad (CAL)$$



- **N Risky Assets: Markowitz Portfolio Optimization**

$$\begin{cases} E(\mathbf{r_{risky}}) = \sum_{i=1}^{n} w_i E(r_i) \\ \sigma^2_{\mathbf{risky}} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \\ \quad = \mathbf{w}^T \Sigma \mathbf{w},\ \Sigma = Covariance\ Matrix \end{cases}$$

$$\mathbf{(FOC)} \quad min_{\mathbf{w}}\ \sigma^2_{risky} \quad s.t.\ E(r_{risky}) = \sum_{i=1}^{N} w_i r_i = R \quad \rightarrow Efficient\ Frontier$$

$$\mathbf{(FOC)} \quad Max_{\mathbf{w}}\ S_{risky} = \frac{E(r_{risky}) - r_f}{\sigma_{risky}} \quad s.t.\ \sum w_i = 1 \quad \rightarrow\ CAL$$

$Another\ Apporach:$

$$Define: \quad \mathbf{R},\ \Sigma,\ \mathbf{w}, \qquad U = r_p - \frac{1}{2} A\sigma_p^2$$
$$Then: \quad R_p = \mathbf{w}^T \mathbf{R}, \qquad \sigma^2_{\mathbf{p}} = \mathbf{w}^T \Sigma \mathbf{w}$$
$$\mathbf{(FOC)} \quad min_{\mathbf{w}}\ U = R_p - \frac{1}{2} A\sigma_p^2$$
$$\rightarrow \quad \mathbf{w} = \frac{1}{A}\Sigma^{-1} R \qquad (best\ allocation)$$

- **Detailed Analysis**

  Define:

$$Average\ Variance\ \ \overline{\sigma} = \frac{1}{n}\sum_{i=1}^{n} \sigma_i^2$$
$$Average\ Covariance\ \ Cov = \frac{1}{n(n-1)} \sum_{j=1, j\neq i}^{n}\sum_{i=1}^{n} Cov(r_i, r_j)$$

  Then, we would have:

$$\sigma_p^2 = \underbrace{\frac{1}{n}\overline{\sigma}^2}_{(firm\ specific\ risk)} + \underbrace{\frac{n-1}{n}\ Cov}_{(systematic\ risk)}$$

  Risk pooling (long-term investment): increase overall risk, increase Sharpe ratio
  Risk sharing: limit pool size, reduce overall risk, increase Sharpe ratio

2.2 Index Model

1. Single Index Model

- **Single Factor Model**

$$r_i = E(r_i) + \beta_i m + e_i$$
$$where\ E(m) = 0,\ E(e_i) = 0$$
$$Characteristics:$$
$$1.\ \sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma^2(e_i)$$
$$2.\ Cov(r_i, r_j) = Cov(\beta_i m + e_i, \beta_j m + e_j) = \beta_i\beta_j\sigma_m^2$$
$$3.\ Corr(r_i, r_j) = \frac{\beta_i\beta_j\sigma_m^2}{\sigma_i\sigma_j} = Corr(r_i, r_m) \times Corr(r_j, r_m)$$
$$4.\ Cov(r_i, r_m) = \beta_i\sigma_m^2, \quad Corr(r_i, r_m) = \beta_i\frac{\sigma_m}{\sigma_i}$$

- **Single-Index Model**

$$\underset{(total\ Risk\ Premium)}{E(R_i)} = \underset{(nonmarket)}{\alpha_i} + \underset{(market)}{\beta_i\ E(R_M)}$$

Regression: $R_i = \alpha_i + \beta_i R_M + e_i$

Security **C**haracteristic **L**ine: $R_i = f(R_M)$

Ajusted Beta $= \frac{2}{3}\hat{\beta} + \frac{1}{3} \times 1$

2. Treynor Black Process

> an active portfolio (A) comprised of the n analyzed securities
> + the market index portfolio / passive portfolio (M), the (n+1)-th assets

$$\begin{cases} \mathbf{E(R_p)} &= \alpha_p + \beta_p \cdot E(R_M) \\ &= (\sum_{i=1}^{n+1} w_i\alpha_i) + (\sum_{i=1}^{n+1} w_i\beta_i) \cdot E(R_M) \\ \mathbf{\sigma_p^2} &= \beta_p^2\sigma_M^2 + \sigma^2(e_p) \\ &= (\sum_{i=1}^{n+1} w_i\beta_i)^2 \cdot \sigma_M^2 + \sum_{i=1}^{n+1}(w_i^2\sigma^2(e_i)) \\ &\rightarrow tracking\ error\ 积极组合的个股风险 \end{cases}, \quad max_{\mathbf{w}}\ S_p = \frac{E(R_p)}{\sigma_P}$$

- **TB procedure**
  - initial position of each security in the active portfolio: $w_i^0 = \frac{\alpha_i}{\sigma^2(e_i)}$
  - scale to summation 1: $w_i = \frac{w_i^0}{\sum_{i=1}^n w_i^0}$
  - alpha of the active portfolio: $\alpha_A = \sum_{i=1}^n w_i\alpha_i$

    residual variance of the active portfolio: $\sigma^2(e_A) = \sum_{i=1}^n w_i^2\sigma^2(e_i)$
  - initial position of the active portfolio: $w_A^0 = \frac{\alpha_A/\sigma^2(e_A)}{E(R_M)/\sigma_M^2}$

beta of the active portfolio: $\beta_A = \sum_{i=1}^{n} w_i \beta_i$

- adjust the initial posistion in the active portfolio: $w_A^* = \frac{w_A^0}{1+(1-\beta_A)w_A^0}$

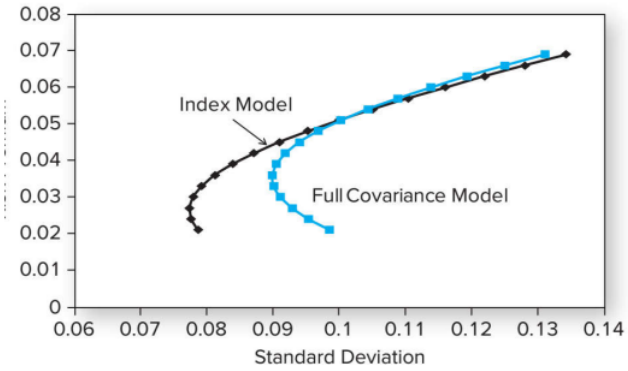
  the optimal risky portfolio: $w_M^* = 1 - w_A^*$, $w_i^* = w_A^* w_i$

risk premium of the optimal risky portfolio: $E(R_p) = (w_M^* + w_A^* \beta_A) E(R_M) + w_A^* \alpha_A$

variance of the optimal risky portfolio: $\sigma_P^2 = (w_M^* + w_A^* \beta_A)^2 \sigma_M^2 + [w_A^* \sigma(e_A)]^2$

Sharp Ratio on the optimal risk portfolio: $S_p^2 = S_M^2 + [\frac{\alpha_A}{\sigma(e_A)}]^2$ ($= IR^2 = \sum_{i=1}^{n} [\frac{\alpha_i}{\sigma(e_i)}]^2$)

- **hedged security return** = $R_i - \beta_i R_M = \alpha_i + e_i$
- **comparison: Index Model vs. Full-Covariance Matrix**



2.3 CAPM

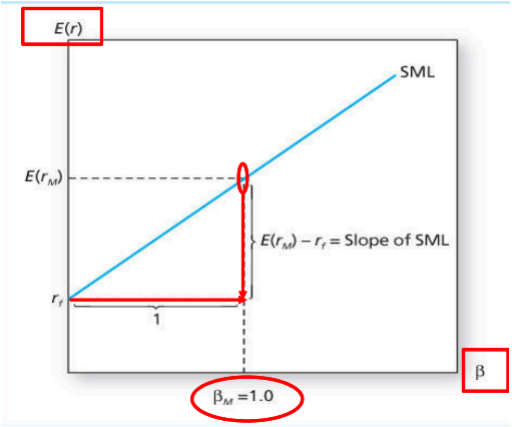
> Market Portfolio (M*): 包括投资者**所有**可交易的证券或资产
>
> $$E(R_M^*) = E(r_M^*) - r_f = \overline{A}\sigma_{M^*}^2$$

Correspondingly, we generate the concept **C**apital **M**arket **L**ine

- **Expected Return - Beta relationship**: $E(R_i) = E(r_i) - r_f = \beta_i (E(r_m) - r_f) = \beta_i E(R_m)$

$$
\begin{aligned}
&market\ return: \\
&E(R_M) = \sum_{i=1}^{N} w_i E(R_i) \\
&market\ portfolio\ risk: \\
&\sigma_{\mathbf{M}}^2 = \mathbf{w}^T \Sigma \mathbf{w} = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j Cov(r_i, r_j) \\
&\quad = \sum_{i=1}^{N} w_i Cov(r_i, \sum_{j=1}^{N} w_j r_j) = \sum_{i=1}^{N} w_i \cdot Cov(r_i, r_M) \ \rightarrow \ risk\ contribution_i \\
&(\mathbf{Equilibrium\ Condition}): E(R_M) = \overline{A}\sigma_M^2 \\
&\qquad \rightarrow \ E(R_i) = \overline{A} Cov(R_i, R_M) = \frac{E(R_M)}{\sigma_M^2} Cov(R_i, R_M) \\
&Define: \qquad \beta_i \equiv \frac{Cov(R_i, R_M)}{\sigma_M^2} \\
&Therefore: \qquad E(R_i) = \beta_i E(R_M)
\end{aligned}
$$

- **S**ecurity **M**arket **L**ine: $E(r) = f(\beta)$



2.4 APT and Factor Model

$$Single\ Factor\ Model:\ R_i = E(R_i) + \beta_i F + e_i$$
$$Single\ Index\ Model:\ R_i = \alpha_i + \beta_i R_M + e_i$$
$$\rightarrow\ CAPM:\ E(R_i) = \beta_i E(R_M)$$

APT: arbitrage if $\alpha_i > 0$

> $$portfolio\ P:\ R_i = \alpha_p + \beta_p R_M, \quad e_p\ diversified\ away$$
> $$\begin{cases} w_p = \dfrac{1}{1-\beta_p} \\ w_M = \dfrac{-\beta_p}{1-\beta_p} \end{cases} \rightarrow\ \beta_z = w_p\beta_p + w_M\beta_M = 0$$
> $$\rightarrow\ \alpha_z = w_p\alpha_p = \frac{1}{1-\beta_p}\alpha_p$$

$$Multifactor\ Model: R_{it} = \alpha_{it} + \sum_{j=1}^{k} \beta_{ij} F_j + \epsilon_{it}$$

$$\rightarrow\ APT:\ E(R_i) = \sum_{j=1}^{k} \beta_{ij} E(R_j^F)$$

Suppose $k = 2$, arbitrage pricing portfolio Q (portfolio A: $E(R_A) = \beta_{A1}R_1^F + \beta_{A2}R_2^F$, if $E(R_A) \neq price(A)$, then we can arbitrage by construting Q)

> For $1 investment, invest:
> β_{A1} to **factor portfolio 1** (with return r_1)
> β_{A2} to **factor portfolio 2** (with return r_2)
> $1 - \beta_{A1} - \beta_{A1}$ to risk free asset (with return r_f)

$$\begin{aligned} Portfolio\ Q:\ E(r_Q) &= \beta_{A1}E(r_1) + \beta_{A2}E(r_2) + (1 - \beta_{A1} - \beta_{A2}r_f) \\ &= r_f + \beta_{A1}[E(r_1) - r_f] + \beta_{A2}E[(r_2) - r_f] \end{aligned}$$

- **Fama-French Three-Factor Model**

$$R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_t$$
$$where \quad SMB \ \rightarrow \ firm\ size, \quad HML \ \rightarrow \ book_to_market$$

3 Frontier Topics

3.1 EMH

Cummulative **A**bnormal **R**eturn

- **Weak Form**: market trading data, technical analysis
  Momentum: positive serial correlation and negative long-term serial correlation
- **Semi-Strong Form**: all publicly available information, fundamental analysis
  Dividend Ratio (D/P): mostly 1% ~ 10%, (+)
  Earning Yield (E/P): P/E mostly 12 ~ 25
  Bond Spread（高低信用评级债的利差）: (+) with market risk premium
  Fama-French Three Factor Model
- **Strong Form**: including private details, insider trading

3.2 Behavioral Finance

- **Cognitive Bias / Heuristics** 认知偏差 / 启发式认知
  Availability bias 可得性偏差：错误的概率预期（忽略样本偏差、先验为主）
  Representativeness 代表性偏差：强加因果/关联，回归效应
  Overconfidence 过度自信：交易frequency越高，平均回报越低
  Conservatism 保守主义：slow react to new information
- **Behavioral Bias / Framing** 行为偏差 / 框架效应
  Convex utility function
  Prospect Theory (Loss Aversion)
  Frame effect (refrence point matters, regret avoidance, mental accounting)
- **Limit to Arbitrage** 有限套利
  noise-trader risk, implementation cost, model risk, sentiment index

Other Brief Notes

1. Bond pricing: Invoice Price = Flat Price + Accrued Interest (in % of par)
2. Callable Bond: (an example)
   Face value (par value): $1,000
   Coupon rate: 6%
   Coupon payment frequency: semi-annual
   Maturity: 10 years
   Call date: After 5 years
   Call price: 105% of face value

3. Determinants of bond saftey
   Coverage ratios: earnings to fixed cost
   Leverage ratios: debt-to-equity
   Liquidity ratios: current asset to current liability
   Profitability ratios
   Cash flow-to-debt ratio
4. Default Bond: recovery rate, default yield spread
5. Interest Rate Uncertainty and Forward Rates
   Forward Rate: future short rates implied from current yield curve
   Expectation Hypothesis: $f_2 = E(r_2)$ (expected future short rate)
   Liquidity premium: $f_n - E(r_n) > 0$, and shall increase over maturity length
6. Synthetic forward loan

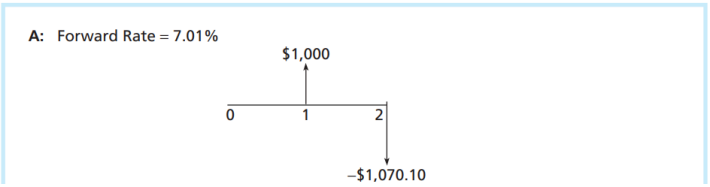
	Initial Cash Flow	In General
Buy a 1-year zero-coupon bond	−952.38	$-B_0(1)$
Sell 1.0701 2-year zeros	+890 × 1.0701 = 952.38	$+B_0(2) \times (1 + f_2)$
	0	0

The initial cash flow (at time 0) is zero. You pay \$952.38, or in general $B_0(1)$, for a zero maturing in 1 year, and you receive \$890, or in general $B_0(2)$, for each zero you sell maturing in 2 years. By selling 1.0701 of these bonds, you set your initial cash flow to zero.[4]

At time 1, the 1-year bond matures and you receive \$1,000. At time 2, the 2-year maturity zero-coupon bonds that you sold mature, and you have to pay 1.0701 × \$1,000 = \$1,070.10. Your cash flow stream is shown in Figure 15.7, panel A. Notice that you have created a "synthetic" forward loan: You effectively *will* borrow \$1,000 a year from now, and repay \$1,070.10 a year later. The rate on this forward loan is therefore 7.01%, precisely equal to the forward rate for the second year.

In general, to construct the synthetic forward loan, you sell $(1 + f_2)$ 2-year zeros for every 1-year zero that you buy. This makes your initial cash flow zero because the prices of the 1- and 2-year zeros differ by the factor $(1 + f_2)$; notice that

$$B_0(1) = \frac{\$1{,}000}{(1 + y_1)} \quad \text{while} \quad B_0(2) = \frac{\$1{,}000}{(1 + y_2)^2} = \frac{\$1{,}000}{(1 + y_1)(1 + f_2)}$$



7. Economic Indicators: leading / coincident / lagging indicators

A. Leading indicators	
1. Average weekly hours of production workers (manufacturing)	Labor Market
2. Initial claims for unemployment insurance	Labor Market
3. Manufacturers' new orders (consumer goods and materials industries)	Supply Chain
4. Fraction of companies reporting slower deliveries	Supply Chain
5. New orders for nondefense capital goods	Supply Chain
6. New private housing units authorized by local building permits	Housing Market
7. Yield curve slope: 10-year Treasury minus federal funds rate	Financial Market
8. Stock prices, 500 common stocks	Financial Market
9. Money supply (M2) growth rate	Financial Market
10. Index of consumer expectations	Survey
B. Coincident indicators	
1. Employees on nonagricultural payrolls	
2. Personal income less transfer payments	
3. Industrial production	
4. Manufacturing and trade sales	
C. Lagging indicators 强调存量 / stock	
1. Average duration of unemployment	
2. Ratio of trade inventories to sales	
3. Change in index of labor cost per unit of output	
4. Average prime rate charged by banks	
5. Commercial and industrial loans outstanding	
6. Ratio of consumer installment credit outstanding to personal income	
7. Change in consumer price index for services	

Industry Analysis: cyclical / defensive industries（防御性行业）

Sensitivity to the Business Cycle:

sensitivity to sales (necessities vs discretionary goods)

operating leverage: the split between fixed and variable costs (higher leverage, more sensitive), **D**egree of **O**perating **L**everage: $DOL = \frac{Percentage\ change\ in\ Profits}{Percentage\ change\ in\ Sales} = 1 + \frac{Fixed\ costs}{profits}$

financial leverage: the use of borrowing (higher interest rate, higher sensitivity)

财务杠杆的比例和CAPM中的β等比例放缩

Sector Rotation: 预计好的行业based on business cycle 的 investment strategy

inflation rate $\uparrow \Rightarrow$ P/E Ratio $\downarrow$ (lowers P and boost nomial E)