Macroeconomics

Catherine

Week 1:
IS-PC-MR Model

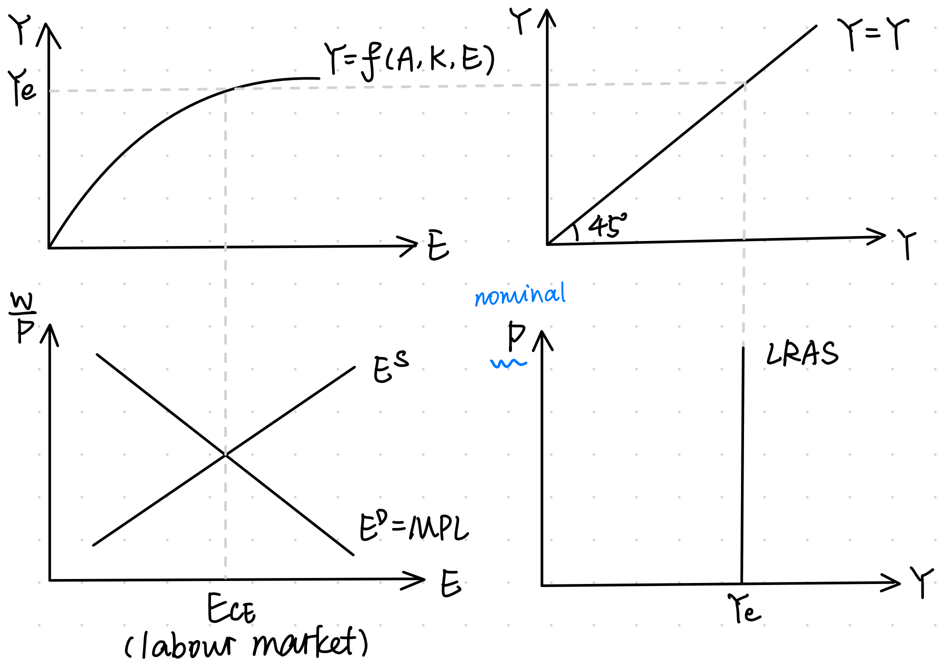
Phillips Curve (imperfect competition equilibrium)
IS curve
Monetary Rule line
the IS-PC-MR Model, Taylor's Rule
extensions (forward-looking, rational expectations)

Phillips Curve

output $Y = f(\underbrace{A}_{\text{technology}}, \overset{\text{capital}}{K}, \underbrace{E}_{\text{labour}})$ A, K fixed over time horizon considered

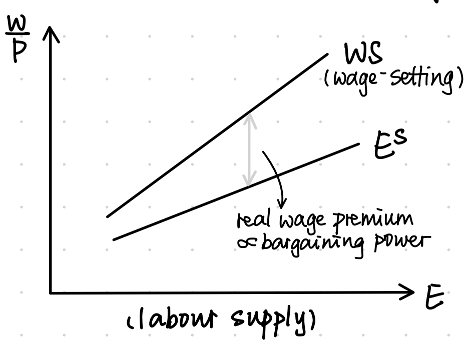
labour market:

- labour demand E^D: marginal product of labour (MPL)
- labour supply E^S: real wage ($\frac{W}{P}$) is opportunity cost of leisure



Labour market under imperfect competition

- finit firms
- finit labours



Trade Unions:

monopoly power ⇒ target $\frac{W}{P} > E^S$

the premium reflects bargaining power

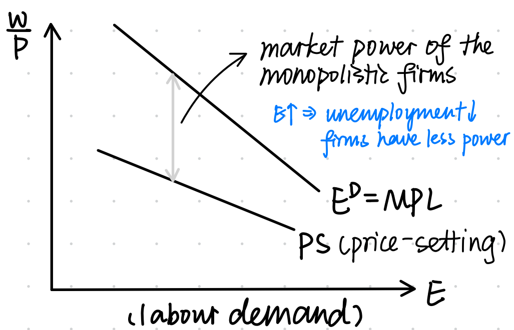
WS diverges from E^S:

union power rises with employment

(fewer unemployed to replace existing workers)

WS curve shifts when ...

1. Bargaining Power
   - legal framework (e.g. restrictions on firing workers)
   - regional / occupational dispersion of employment
     unemployed labour concentrated in one region / industry ⇒ limits to labour mobility
2. Aspirations of trade unions
   - labour productivity (union may share in the gains from rising output per worker)
   - wage accords (union may limit wage premium in return for public investment in their industry and improved working conditions)
3. Demographic factors
   labour supply (net migration, retirement ages)
   (however, normally not proportionately: new workers not perfectly substitutable, $E^S \Rightarrow$, WS→)



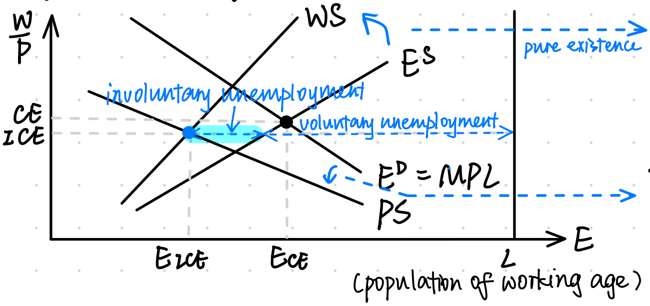
Firms with monopoly power:
wedge between MPL and PS
= MPL not offered to workers
= basis for excess profits by firms

PS → horizontal:
k<0: profit share counter-cyclical (falls with employment)

PS curve shifts when ...

1. Monopsony power
2. Other agents' claims of firms
   - other factors raise relative price (e.g. 1970s OPEC increased $price^{oil}$↑ ⇒ western $\frac{W}{P}$ stagnated)
   - governments' claim on MPL through payroll taxes (tax↑ ⇒ PS↓)
3. Consistence with E^D (MPL)
   a rise in productivity ⇒ PS↑
   (but extent of PS shift limited if firms use rise in A as opportunity to raise profit share)

Equilibrium of labour market (imperfect competition)



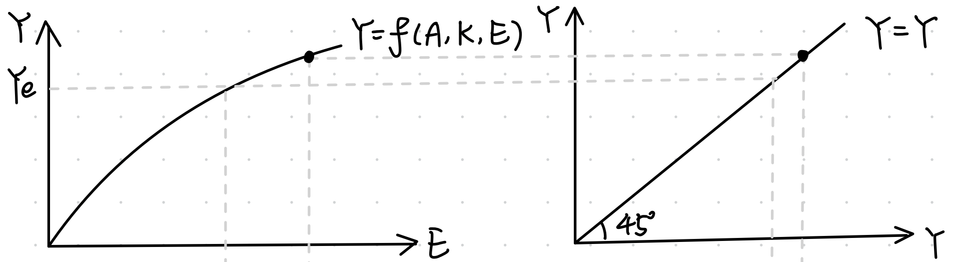
the reason for involuntary unemployment (imperfect competition in labour supply)

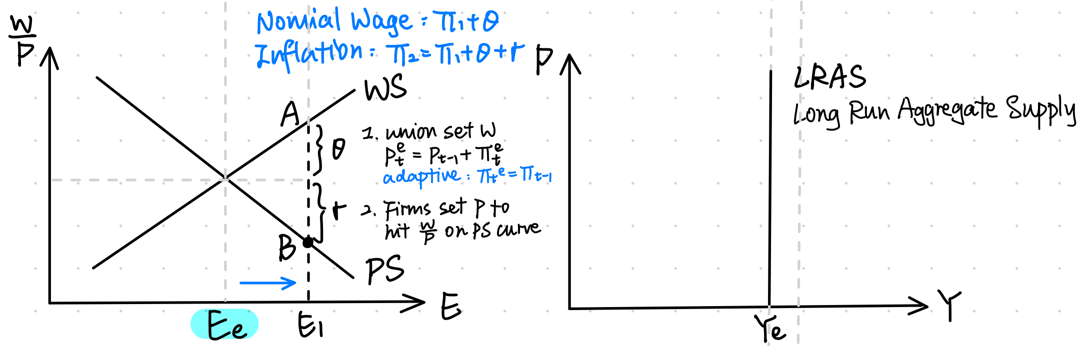
the extent to which involuntary unemployment happens (imperfect competition in goods market)

Except when WS // E^S

efficiency wages ⇒ WS curve

Equilibrium under imperfect competition





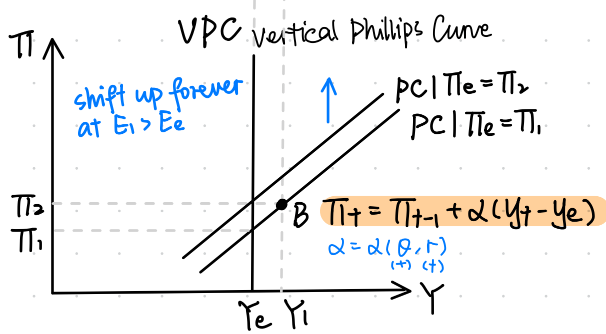
NAIRU (Non-Accelerating Inflation Rate of Unemployment)

Phillips Curve:

Originally link Π to u,

modern macro extends to (Y, Π)

(external demand shock raises employment to E_1)



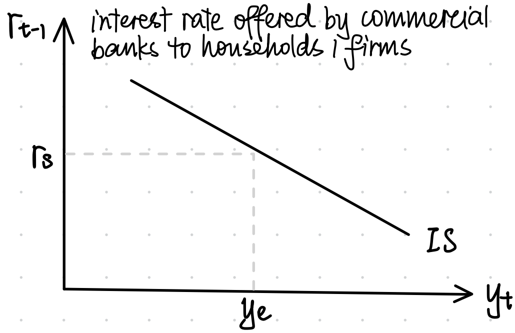
IS Curve

output = f(interest rate) : $y_t - y_e = -a(r_{t-1} - r_s)$

equilibrium output — stabilizing (equilibrium) real interest rate

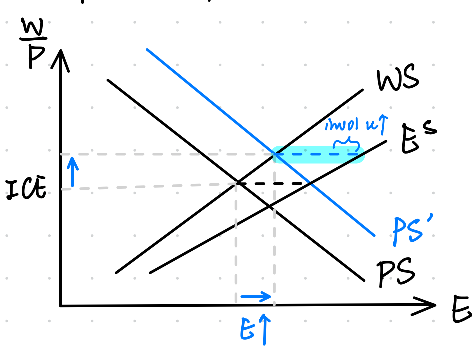
a: interest sensitivity of expenditure

r_{t-1}: 1 period lag in $r \rightarrow y$ transmission, e.g. BoE suggests 12-18 month



CB set nomial interest rate (offered to commercial banks)
⇒ set r^P_{t-1} ⇒ transmit to r_{t-1}
⇒ influence output (via IS) and inflation (via PC) in t

A positive productivity shock:



involuntary ↑, voluntary ↓, total ↓

an increase in producer market power : ↓↑↑

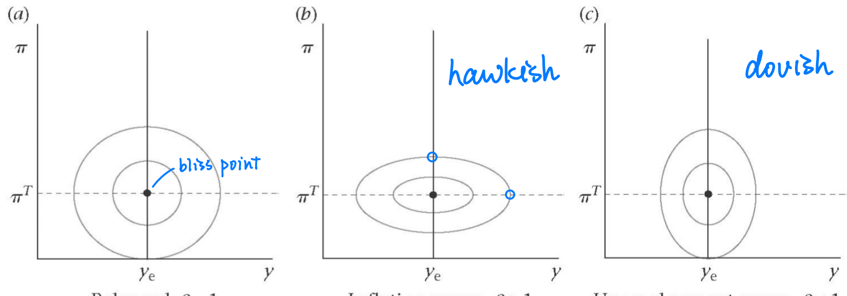
the abolition of employment protection legislation : ↓↑↓

MR (Monetary Rule)

CB's objective function (quadratic deviations from (y_e, π^T)):

$$L = (y_t - y_e)^2 + \beta(\pi - \pi^T)^2$$

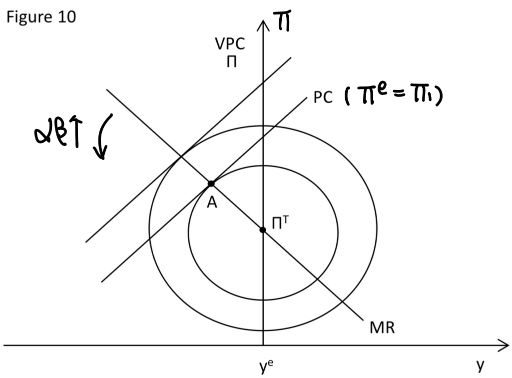
1. convex preferences over deviations
2. β measures aversion to π deviations versus y deviations
3. loss contours defined around bliss point (y_e, π^T)



Balanced: $\beta = 1$ — Inflation averse: $\beta > 1$ — Unemployment averse: $\beta < 1$

CB minimizes loss function s.t. PC constraint

Figure 10



Solution: tangency point between PC and loss contour

MR: set of best responses for all possible π^e ⇒ monetary rule

$$\min_y L = (y_t - y_e)^2 + \beta(\pi_t - \pi^T)^2$$

$$\text{s.t. } \pi_t = \pi_{t-1} + \alpha(y_t - y_e)$$

$$\Rightarrow \pi_t = \pi^T - \frac{(y_t - y_e)}{\alpha\beta}$$

$\beta \uparrow$ ⇒ MR rotated counter-clockwise

more inflation averse CB eliminates excess inflation more quickly

high β comes from: past experience of high inflation
openness to trade/finance which makes high π more distorting (e.g. Swiss National Bank)

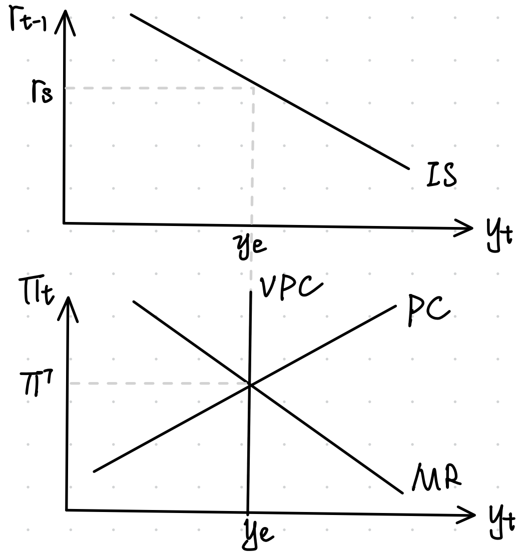
$\alpha \uparrow$ ⇒ MR rotated counter-clockwise

steeper PC ⇒ inflation controlled more efficiently ⇒ CB responds with rapid π reduction

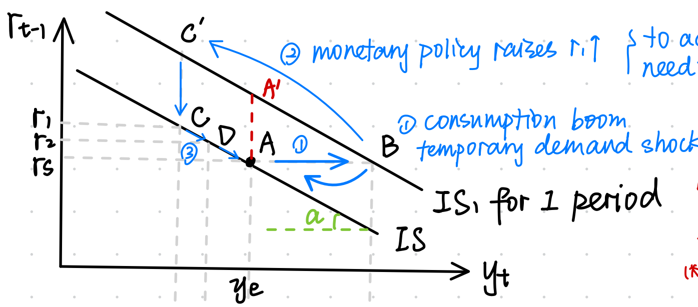
determinants of α

IS-PC-MR Model

Equilibrium:

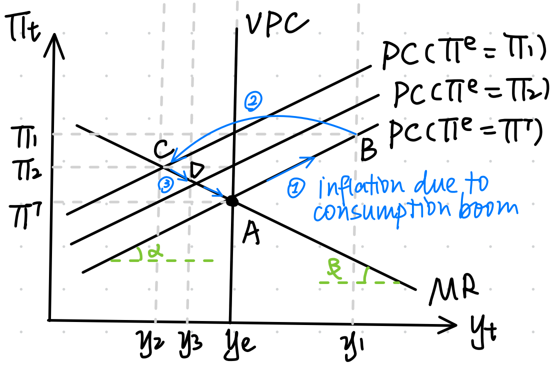


unexpected, temporary positive demand shock:



{ to achieve C next period on PC
need to raise r_1 today

if demand shock forecasted,
set A' when shock occurs
⇒ y and π never deviates
(*) However, uncertainty limits CB



(*) real interest rate (r_1)
peak in period in which
shock occurs
⇒ nominal interest rate?

$i_1 = r_1 + \pi^T$

$i_2 = r_2 + \pi_1$

($i = r + \pi^e$, where π^e
is backward-looking)

PC($\pi^e=\pi_1$): $\pi_{t+1}=\pi_t+\alpha(y_{t+1}-y_e)$

MR(t+1): $\pi_{t+1}=\pi^T-\frac{(y_{t+1}-y_e)}{\alpha\beta}$

IS(original): $y_{t+1}-y_e=-a(r_t-r_s)$

$\Rightarrow$ solution for the peak interest rate: $r_t-r_s=\frac{1}{a(\alpha+\frac{1}{\alpha\beta})}(\pi_t-\pi^T)$

$r_t-r_s=f(\underset{(-)}{a},\underset{(+)}{\beta},\underset{(?)}{\alpha})$

1. $a\uparrow$: monetary transmission more powerful via IS
   $\Rightarrow$ raise r by less (flatter IS)
   large a in UK (high fraction of floating rate mortgages, high private sector debts), which is a central economic argument against entry into € in 1990s.
2. $\beta\uparrow$: inflation aversion$\uparrow$ $\Rightarrow$ quickly close inflation deviation
   different inflation tolerance $\Rightarrow$ inefficient common monetary policy in Eurozone
3. effect of α: ambiguous
   $\alpha\uparrow$: flatten out monetary policy rule (MR)
   c closer to the bliss point
   $\alpha^2\beta>1$: MR starts off relatively flatter $\Rightarrow$ $\alpha\uparrow$ lowers r_t-r_s, lucky!

Taylor's Rule

plug $a=\alpha=\beta=1$ in $r_t-r_s=\frac{1}{a(\alpha+\frac{1}{\alpha\beta})}(\pi_t-\pi^T)$:

$r_t-r_s=\frac{1}{2}(\pi_t-\pi^T)$

Talyor's Rule: $r_t-r_s=\frac{1}{2}(\pi_t-\pi^T)+\frac{1}{2}(y_t-y_e)$

$$\underbrace{i_t}_{\text{nomial}}=(\underbrace{r^*}_{\text{target}}+\underbrace{\pi^*}_{\text{target}})+0.5(\underbrace{y_t}_{\text{actual}}-\underbrace{y_t^*}_{\text{potential}})+1.5(\underbrace{\pi_t}_{\text{current inflation}}-\pi^*)$$

(based on a constant growth assumption)

close to 2 when estimated using real time data

$i_t-\pi_t=r^*+0.5(y_t-y_t^*)+1.5\pi_t-1.5\pi^*-\pi_t+\pi^*$

$\Rightarrow r_t-r^*=0.5(y_t-y_t^*)+0.5(\pi_t-\pi^*)$

Why output gap included:

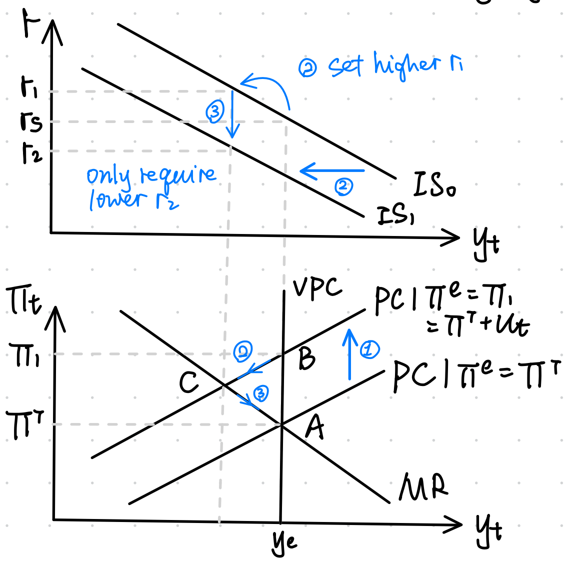
In general PC assumption, PC has no lag from $y\to\pi$. So output gap doesn't convey any information over and above inflation gap
$\Rightarrow$ reacting to inflation alone sufficient
However, if it takes 1 period for current output gap to impact inflation then current output gap will reveal there has been an IS shock even through inflation gap is zero and it then makes sense to react to output gap quickly.

Phillips Curve shocks

PC augmented with random shock term:

(given $\pi_{t-1} + \alpha(y_t - y_e)$, inflation may be raised by cost-push shocks, e.g. higher import prices)

$$\pi_t = \pi_{t-1} + \alpha(y_t - y_e) + u_t$$



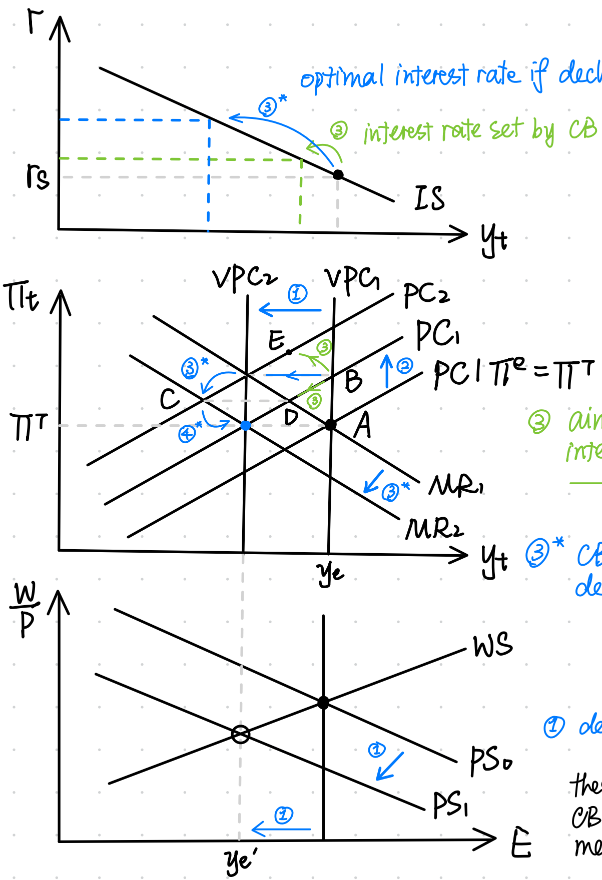
UK in 2008

1. unexpected 1 period positive cost-push shock that shifts PC up ⇒ r_1
   (rising energy / food prices)
2. negative IS shock ⇒ r_2
   (falling expenditure in aftermath of credit crunch)

the Great Inflation (1970s)

—— UK inflation peaked at over 20% per annum in 1970s

Orphanides and other writers attribute episode to declines in productivity which were not immediately recognized by CBs, resulting in policy errors that raised inflation.



optimal interest rate if decline in productivity is observed

③ interest rate set by CB unaware of productivity change

② Phillips Curve shock

③ aim at D along MR_1 given that CB interprets this as pure cost-push shock
—— ended up at E

③* CB fully informed of productivity declines, move $MR_1 \rightarrow MR_2$

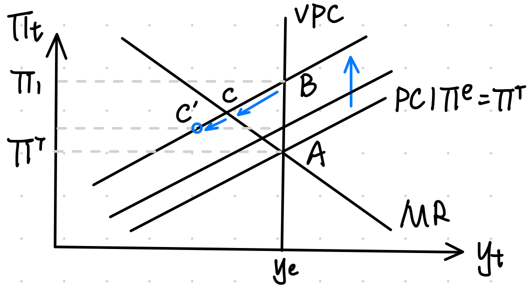
① decline in labour productivity

these monetary policy errors repeat until CB inferred that rising inflation despite $y < y_e$ meant productions have declined

Forward-Looking CB

Simplest inter-temporal loss function:

$$L_t = (y_t - y_e)^2 + \beta(\pi_t - \pi^T)^2 + \frac{1}{1+\delta}\left((y_{t+1} - y_e)^2 + \beta(\pi_{t+1} - \pi^T)^2\right)$$



deviation $C \rightarrow C'$:

$$MC_t - MB_t = \frac{1}{1+\delta}(MB_{t+1} - MC_{t+1})$$

More general loss function:

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+\delta}\right)^i L_{t+i}$$

previous solution for the peak interest rate: $r_t - r_s = \frac{1}{a(\alpha + \frac{1}{\alpha\beta})}(\pi_t - \pi^T)$

Now: δ enters as further determinant of inflation reaction

smaller $\delta \Rightarrow$ bigger deviation from "c"

since 1970s, CB more vigorous policy responses to inflation disequilibrium since monetary policy control has shifted from elected governments to independent CBs who typically have longer horizon (smaller δ applicable)

Rational Expectation

Muth: rational expectations are 'model consistent expectations'

Phillips Curve under RE:

$$\pi_t = E_{t-1}\pi_t + \alpha(y_t - y_e)$$

$$\Rightarrow y_t - y_e = \frac{1}{\alpha}(\pi_t - E_{t-1}\pi_t)$$

$\pi^e = \pi^T$ validated $\Rightarrow$ after shock path is $A \rightarrow B \rightarrow A$: painless disinflation.

1. only case in which CB might undertake stabilizing monetary policy is if it could foresee a shock that private sector does not.
   (but why not simply release information to private sector)
   blips in inflation from one-off rises in price level (e.g. ↑VAT) ⇒ constant expectations
2. (Limits) Non-RE agents exert more than proportionate effect on outcomes, as RE would recognize the existence of non-RE.
3. (Limits) Require CB's credibility and other agents' rationality
4. (Limits) Sticky prices and wages prevent immediate recovery to A

CB survey π^e in policy decisions : Macro RE dominant paradigm

Week 2:
Monetary Policy

Inflation Bias
Sticky Prices
NKPC, inflation dynamics, optimal monetary policy

Inflation Bias

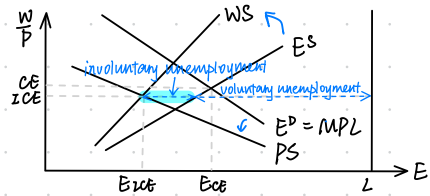
Definition: bias refers to equilibrium inflation in excess of optimal inflation target.

Reasons for inflation bias

CB loss function amended to: $L = (y_t - y^T)^2 + \beta(\pi - \pi^T)^2$, $(y^T > y_e)$

— Bliss level of output y^T > feasible level y_e on VPC

1. E sub-optimal at y_e level because MPL > opportunity cost of leisure (a social planner seeks $E^S = E^D$ (E_{CE}), efficient employment level in presence of distortions from imperfect competition)
2. $y^T > y_e$ could be from political cycle (raising GDP seen as signal of economic competence and a pre-election vote winner)



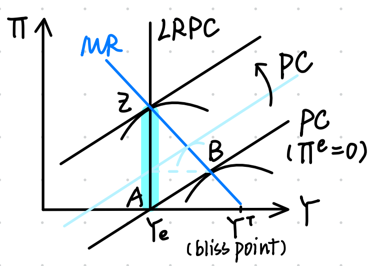
$\min_y L = (y_t - y^T)^2 + \beta(\pi_t - \pi^T)^2$ s.t. $\pi_t = \pi_{t-1} + \alpha(y_t - y_e)$

FOC: $(y_t - y^T) + \alpha\beta(\pi_t - \pi^T) = 0 \Rightarrow \pi_t = \pi^T - \frac{(y_t - y^T)}{\alpha\beta}$

$\Rightarrow (y_t = y_e)$ plug in: $\pi_Z = \pi^T + \frac{y^T - y_e}{\alpha\beta}$

- international differences in macro parameters $\alpha, \beta, y^T - y^e$ predict differences in inflation outcomes
- Romer (1993): greater trade openness (import share in domestic consumption) increases α and therefore reduces equilibrium π.
  mon pol loosened ⇒ exchange rate depreciates ⇒ raises import prices
  Therefore, import prices adjust more quickly than domestic prices ⇒ $\alpha\uparrow$

1. Equilibrium with Adaptive Expectation



Static Loss: $A \to B \to \cdots \to Z$

Dynamic Loss: $\min \sum_{i=0}^{\infty}$

polar case $\delta = 0$: never deviate from A

case of static loss ($\delta \to \infty$): $A \to B \to \cdots \to Z$

intermediate values of δ: less than full bias at Z

2. Equilibrium with Rational Expectation (More fundamental)

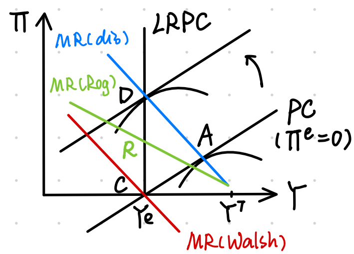
only output / inflation outcome that can occur is Z, even under dynamic loss

— CB policy proposal to achieve A is time inconsistent

Solution: delegation

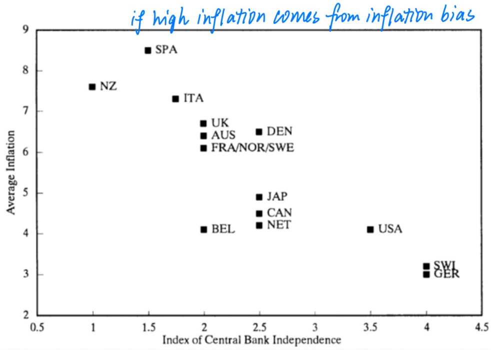
opportunity to reset preferences of policy-makers.

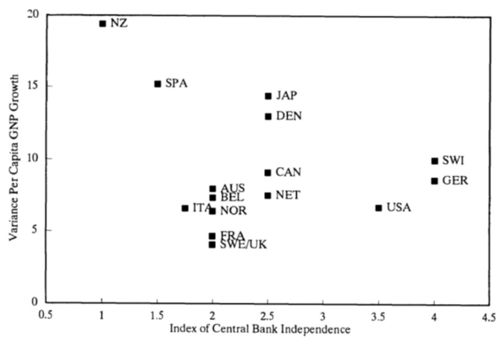
1. most straightforwardly, replace y^T with y_e
   but: ① easier said than done ② y_e may be over-estimated due to productivity changes
2. Rogoff: delegation to conservative central banker
   ⇒ same y^T, larger β ⇒ flatter MR
   (Con): ① no full elimination
   ② higher sacrificed output after a cost-push shock to eliminate inflation
   ③ only effective if rational expectation of the new CB loss function
   (otherwise, inertial ⇒ PC($\pi^e = \pi^2$), than any changes < staying at π^2, they may prefer not to appoint conservative CB in the first place)
3. Walsh's approach
   CB penalized for each unit increase in $\pi > \pi^T$ (through CB dismissal or cut in CB resources)
4. Svensson
   do nothing other than setting an inflation target below π^T



Central Bank Independence and Inflation Performance

if high inflation comes from inflation bias





(who is represented on the board, for how long does a board member serve, is the CB obliged to lend to the government in all circumstances)

Alessina and Summers (1993): low π achieve by ↑CBI → no additional volatility in real variables

Sticky Prices

Throughout analysis in previous lectures flexible prices assumed.

- Keyenesian: adaptive expectation, flexible prices
- New Classical: rational expectation, sticky prices

Retain rational expectations (New bit) but restore Keyenesian outcomes in models in which prices sticky in short-term so markets do not clear and output / employment fluctuate instead.

Why sticky prices? consequence: firms not setting optimal prices, monopolists

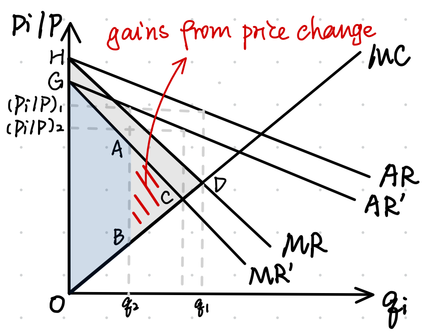
menu costs: physical/time cost of new price labels; customer goodwill; disruption to financial planning

— challenges: online pricing; how small menu costs could contribute to aggregate price inertia?

Ball, Mankiw and Romer (BMR) 1988

Set-up

- imperfect competition amongst firms
- rational expectations
- firms plan to re-set prices at fixed intervals (time contingent pricing)
  — thus no marginal menu costs (sunk costs)
  (but at all other points there is an incremental menu cost to be paid for unscheduled price change)
- uniform distribution of scheduled price adjustments



prior to the shock: HDO
after the shock: GABO (sticky price)
GCO (flexible price)
Best: HDCG (all firms cut price)

take into account ABC when assess cost of leaving prices unchanged
— coordination failure

Condition (ABC < menu costs < HDBAG) most likely satisfied:
when MC relatively elastic (flatter MC)
(this reduces size of private profit gain ABC relative to external gain HDCG)

Real Rigidities $\xrightarrow{\text{contribute to}}$ flat MC

Implications:

1. PC steeper if planned price reviews more frequent.
   interval = 12 months. $\frac{1}{12}$ firms review on Jan / Feb ...
   interval = 6 months: $\frac{1}{6}$ firms review on Jan / Feb ... ⇒ more price flexibility
2. planned price reviews more frequent in regimes of higher average inflation
   ⇒ countries with higher historic inflation face steeper PC.

Dynamic Adjustment:

- Extensions of BMR framework show time required for adjustment can exceed 12 months (or other fixed calendar time) in which all firms have opportunity to vary prices
  – why? when a demand shock occurs the 1/12 firms free to adjust know whether menu cost means other firms will hold prices fixed
  – if it does then the flexible price firms know that by varying prices they are changing their relative price
  – aversion to changes in relative prices (e.g. fear of sparking price war) may mean firms act gradually, e.g. in Fig 23 lower price only part of way from $(p_i/p)_1$ to $(p_i/p)_2$
  – when next set of firms get chance to costlessly adjust prices they know other firms implemented only partial adjustment and respond in kind
  – so aggregate price adjustment can be protracted 持久的

Micro-foundations for sticky prices

1. empirical evidence: significant intervals between price adjustments (though vary by sector, e.g. goods vs. service)
2. real rigidities

   menu cost frictions will bite when there are significant real rigidities shaping balance between private and external benefits from price adjustment, but real rigidities from elastic labour supply rarely observed in practice (most estimates suggest relatively inelastic labour supply - same problem as that confronting RBC theory)

3. If we think menu costs as fixed costs:
   for large shocks even private profit triangle ABC will expand to exceed menu costs

New Keynesian Phillips Curve

Calvo (1983)

Each firm can freely set its current price with probability λ (and probability $(1-\lambda)$ price stuck at level from last period)

Let inflation rate amongst flex-price firms be π^*. Given inflation amongst other firms is zero, aggregate inflation rate in t is:

$$\pi_t = \lambda \pi_t^*$$

1. π_t^* is function of $\pi_t + \alpha(y_t - y_e)$
2. In setting π_t^*, flex-price firms must also take into account $(1-\lambda)$ probability that π_t^* defines price level they must live with in $t+1$.
   $\Rightarrow$ account for $E_t\pi_{t+1} + \alpha E_t(y_{t+1} - y_e)$
   (similarly, there is a $(1-\lambda)^2$ probability π_t^* should take account of $E_t\pi_{t+2} + \alpha E_t(y_{t+2} - y_e)$ ...)
   $\Rightarrow \pi_t^* = \sum_{i=0}^{\infty}(\frac{1}{1+\delta})^i (1-\lambda)^i (E_t\pi_{t+i} + \alpha E_t(y_{t+i} - y_e))$
   $\pi_t = \lambda\sum_{i=0}^{\infty}(\frac{1}{1+\delta})^i (1-\lambda)^i (E_t\pi_{t+i} + \alpha E_t(y_{t+i} - y_e))$ (1)
   $\pi_{t+1} = \lambda\sum_{i=1}^{\infty}(\frac{1}{1+\delta})^{i-1} (1-\lambda)^{i-1} (E_{t+1}\pi_{t+i} + \alpha E_{t+1}(y_{t+i} - y_e))$ (2)
   (1) $- \frac{1}{1+\delta}(1-\lambda)\cdot$ (2):
   $\pi_t - \frac{1-\lambda}{1+\delta}E_t\pi_{t+1} = \lambda\pi_t + \lambda\alpha(y_t - y_e)$
   $\Rightarrow \pi_t = \frac{1}{1+\delta}E_t\pi_{t+1} + \frac{\alpha\lambda}{1-\lambda}(y_t - y_e)$
   or: $\pi_t = E_t\pi_{t+1} + k(y_t - y_e)$
   assuming $\delta = 0$ and defining $k = \frac{\alpha\lambda}{1-\lambda}$

AEPC: $\pi_t = \pi_{t-1} + \alpha(y_t - y_e) + u_t$
REPC: $\pi_t = E_t\pi_{t+1} + \alpha(y_t - y_e) + u_t$
NKPC: $\pi_t = E_t\pi_{t+1} + k(y_t - y_e) + u_t$, $k = \frac{\alpha\lambda}{1-\lambda}$
(+ sticky price)

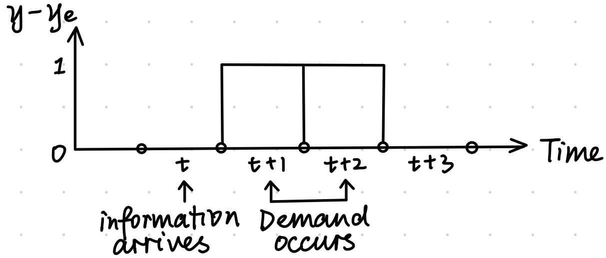
Structure of NKPC

1. PC under RE: $\pi_t = E_{t-1}\pi_t + \alpha(y_t - y_e)$
   NKPC (sticky price): $\pi_t = E_{t+1}\pi_t + \alpha(y_t - y_e)$
2. Equivalence of NKPC:
   $\begin{cases} \pi_t = E_t\pi_{t+1} + k(y_t - y_e) \\ \pi_{t+1} = E_{t+1}\pi_{t+2} + k(y_{t+1} - y_e) \end{cases} \Rightarrow E_t\pi_{t+1} = E_t\pi_{t+2} + E_t k(y_{t+1} - y_e)$
   $\Rightarrow \pi_t = E_t\pi_{t+2} + E_t k(y_t - y_e) + k(y_t - y_e)$
   $\cdots \Rightarrow \pi_t = E_t\pi_{t+\infty} + E_t\sum_{i=0}^{\infty}k(y_{t+i} - y_e)$ plug in $E_t\pi_{t+\infty} = \pi^T$
   $\Rightarrow \pi_t = \pi^T + E_t\sum_{i=0}^{\infty}k(y_{t+i} - y_e)$

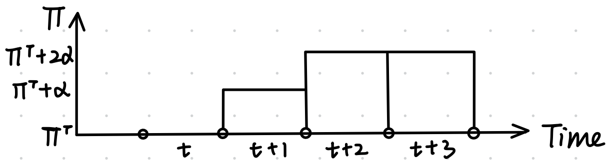
support: CB will do whatever it can do to make the inflation return to its target

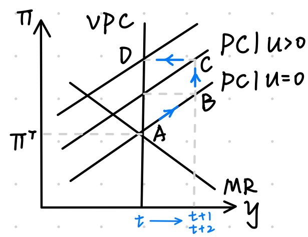
NKPC's prediction

1. anticipated demand expansion
   suppose initially $y = y_e$, $\pi = \pi^T$
   In period t information arrives so that $E_t y_{t+1} - y_e = E_t y_{t+2} - y_e = 1$, $E_t y_{t+3} - y_e = 0$ (and periods thereafter)

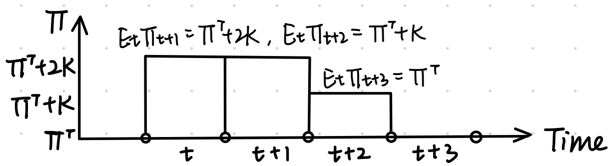


Inflation under AEPC: $\pi_t = \pi_{t-1} + \alpha(y_t - y_e)$





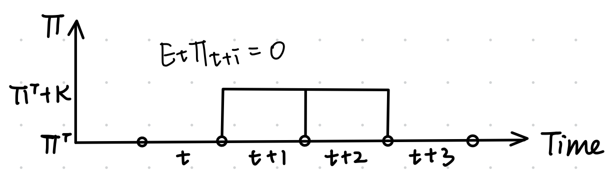
Inflation under NKPC: $\pi_t = E_t \pi_{t+1} + k(y_t - y_e) = \pi^T + E_t \sum_{i=0}^{\infty} k(y_{t+i} - y_e)$



current output gap predicts expected change in inflation of opposite sign
$E_t \pi_{t+1} - \pi_t = -k(y_t - y_e)$

$\begin{cases} \text{AEPC: inflation persistence satisfied} \\ \text{NKPC: inflation adjusts rapidly like a `jump variable'} \end{cases}$

2. unexpected output shocks
   Inflation under AEPC: nothing changes
   Inflation under NKPC:



the correlation between output gap and inflation is now positive

Hybrid Phillips Curve (Roberts, 1995)

$$\pi_t = \tau \pi_{t-1} + (1-\tau) E_t \pi_{t+1} + k(y_t - y_e)$$

Applied Research suggests $\tau \approx 0.5$

Phillips Curve Models: Summary of Key Features

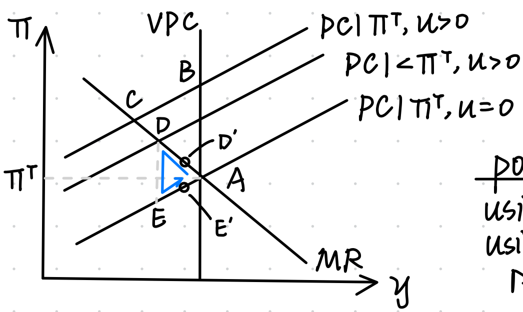
Model	Optimising Microfoundations?	Rational Expectations?	Price Level Persistence?	Inflation and Output Persistence?	Positive Sacrifice Ratio?
Roberts	Yes	No	Yes	Yes	Yes (but can be perverse)
Calvo*	Yes	Yes	Yes	No	Yes (but can be perverse)

AEPC: $A \to B \to\to A$
REPC: $A \to B \to A$
NKPC: $A \to D \to\to A$

NKPC and optimal monetary policy

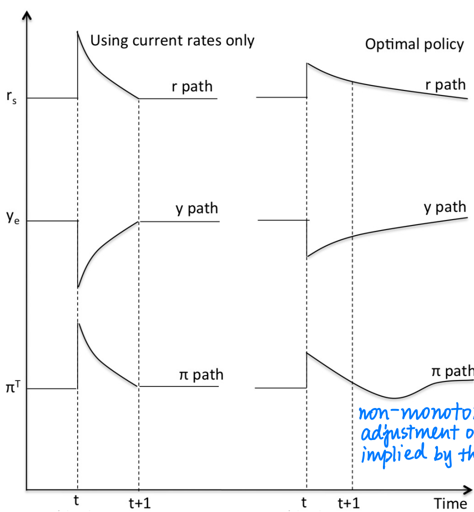
NKPC augmented with cost-push shock: $\pi_t = E_t \pi_{t+1} + k(y_t - y_e) + u_t$

Assume:
1. 1-period unexpected cost-push shock: $u_t > 0$
2. no lag in IS curve so that $r_t \to y_t$



tradeoff between π and y:
- demand shock: No
- supply shock: Yes

policy paths	t	t+1	t+2	t+3
using r_t only	C	A	A	A
using r_t, r_{t+1}	D	E	A	A
r_t, r_{t+1}, r_{t+2}	D'	E'	E'	A



non-monotonic adjustment of π implied by the PP target

However, optimal policy is time inconsistent.

in t+1, prefer A to E
rational agents deduce this

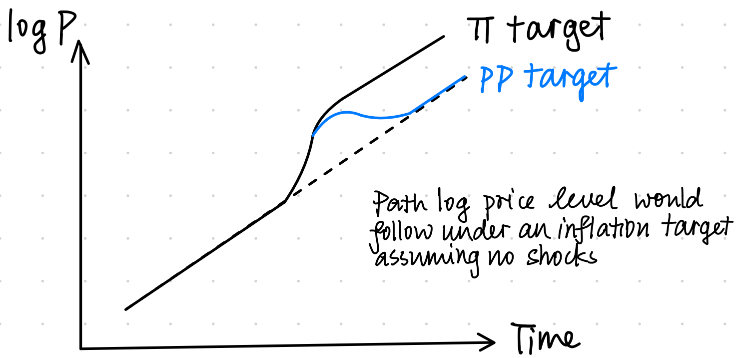
⇒ **stabilization bias**

(CB is forced to over-stabilize economy through raising current rates by more than is optimal because of its inability to commit to future interest rate outcomes)

Modification ⇒ optimal policy time consistent

— need to modify CB objectives such that points such as E in t+1 are preferred to A

Answer: a price path target ($\pi = \frac{dP/P}{dt} = \frac{d(\log P)}{dt}$)



Week 3:
Macroeconomics at ZLB

Macro facts after 2008
Zero Lower Bound (ZLB)
- Forward Guidance (with inflation target)
- PP targets, Nominal GDP targets
- Quantitative Easing

Weakness in Macro since 2008

In UK: unemployment, average real wages, productivity, GDP
- 2008: ↓7% in output (in less than 18 months at start of recession)
- 2008-2011: flat output
- 2013: 14% smaller than projected in 2008 (historical growth rate)
- 2016: recover to 2008 peak

US: faster GDP recovery but weaker unemployment recovery
— depth and breadth of downturn labelled the 'Great Recession'

Keynesian view

lenders hit in US sub-prime mortgage market
amplified by failure of Lehman's in 2008
European sovereign bond crisis start since 2010
} ⇒ credit crunch

— A pool of savings that could not be recycled as expenditure due to lack of bank lending or other financial intermediation

⇒ demand deficit that impacted IS curve and GDP

Alternative view: financial crisis left scar of lower equilibrium output
(decline in supply from impaired capital stock, resource mis-match, tech regress in sectors like financial services)

Monetary Policy since 2008

1. large interest rate cuts
   - BoE cuts policy rate by 150 basis points in Nov. 2008
   - by 2009 central bank interest rate at 0.5% in UK, 0~0.25% in US, 1% in Euro area
2. ECB started raising rates in 2011 but quickly reversed track and maintain policy rate at 0.05% in late 2014 (historically low)
   - 0.05% = a lending rate to commercial banks
   - a short-term deposit rate offered by the ECB is slightly negative
   - BoJ policy interest rate stuck at 0.1% for over a decade

Zero Lower Bound (ZLB)

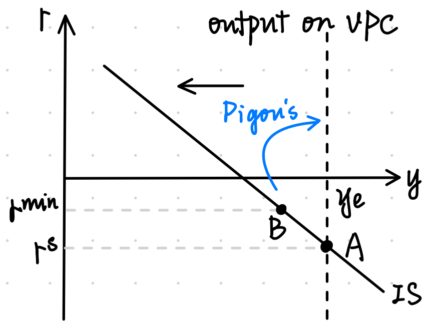
a lower bound in slightly positive territory (CB losses which weaken legitimacy of CBs)

CB policy rates: rates at which CB will lend short-term to commercial banks
CB deposit rates: rates at which commercial banks can invest funds overnight
↳ lower bound just below zero (cost of storage/insurance)

$\Rightarrow r^{min} = -\pi^T$

Problem for macro stabilization

For large negative IS shocks could observe $r^s < r^{min}$ (lowest real interest rate policy can effect is insufficient for equilibrium output



ideal equilibrium A impossible
realizable equilibrium B unstable:

$y < y_e \Rightarrow \pi \downarrow \Rightarrow E(\pi) \downarrow$ { gradual under AE / rapid under RE

$\Rightarrow r^{min} = -\pi \uparrow \Rightarrow y \downarrow$ (deflation)

faster adjustment of prices ($\alpha \uparrow$),
faster adjustment of expectations (more RE agents),
$\Rightarrow$ faster the deflationary spiral
—— Krugman: curse of flexibility

Economy fails to equilibrate by itself: Keyne's liquidity trap
ZLB is an instance

1. Pigou: falling prices ultimately stabilize
   $P \downarrow \Rightarrow$ real value of private sector assets $\uparrow$ (fixed nominal value)
   $\Rightarrow$ positive wealth effect on C $\Rightarrow$ shift IS right faster

   Fragility: reliance on sufficiently large stock of private assets
   in debtor economies, falling prices could raise the real value of debts and cause IS to shift left whilst r^{min} rising (negative wealth effect)

2. Empirical study: hysteresis (滞后) in output in deep recessions
   then ye adjusts downwards in the direction of actual output
   - capital scrapping — unprofitable firms suspend investment and allow depreciation to take effect (PS shifts left $\Rightarrow y_e \downarrow$)
   - long-term unemployed stop seeking work (WS shifts left $\Rightarrow y_e \downarrow$)

   $\Rightarrow$ the economy never recovers to its original ye level

Optimal policy at ZLB: forward guidance

keep rates at ZLB for greater length of time (forward guidance)

⇒ current long-term rates $\underbrace{i_t^n}_{\text{n-period interest rate}} = \dfrac{i_t + E_t i_{t+1} + \dots + E_t i_{t+n-1}}{n} + \underbrace{\theta_t^n}_{\text{risk premium (n periods)}}$

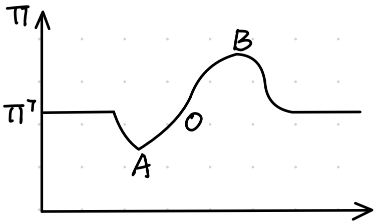
1. lower long rates: cheaper financing for banks, ultimately for firms
2. shift IS curve right (IS curve: a function of real short rate)

How far does forward guidance have to go?

1. imparts stimulus to economy, but fails to eradicate demand deficit
2. "commit to irresponsibility"
   CB must pledge to keep policy rates at ZLB not only until economy equilibrates but until inflation exceeds the official target
   ⇒ $\underbrace{\pi^e_{t+n|t}}_{\text{expected inflation between t and t+n}} \uparrow$, $r_t^n \downarrow = i_t^n - \pi^e_{t+n|t}$
3. Empirical study:
   Fed vowed to keep rates at ZLB for an extended period after full employment reached
   BoE linked its original forward guidance to inflation forecasts two years out below 2.5%
   BoJ explicitly committed to inflation above 2% inflation (2016 November)
4. Time inconsistency
   CB breaks promise once inflation comes back to 2%
   Private agents do not act upon forward guidance
   - interest group pressure typically comes to bear forcing an early exit from super loose monetary policy (e.g. accusations of currency manipulation, pensioners' savings being taxed away)
   - right-wing economists: low rates induce asset price bubbles, keep alive zomby firms
   - in practice, CBs tend to withdraw stimulus before $\pi > \pi^T$
     —— a factor prolonging Great Depression in US (1930s), Japanese 'Lost Decade' (21C)

PP targets

Restoring price path: raise π to point B above π^T before it can stabilize at long-run target 2%



$\Rightarrow$ lock lower long-term real rates
induce expenditure necessary for $y = y_e$

advocated by Greg Mankiw

Pro: 1. automatic stabilizer that lowers real interest rate as soon as inflation starts to fall
2 ward off demand deficiency

inflation target: delay in scheduling monetary stimulus (internal tensions)
$\Rightarrow$ adverse demand multipliers set in ①
—— asymmetric target view:
{ quick to snuff out $\pi > 2\%$
relaxed about $\pi < 2\%$, allow it to equilibrate gradually
$\Rightarrow$ real income transferred from borrowers to savers
$\Rightarrow$ consumption propensity $\downarrow$ $\Rightarrow$ $D\downarrow$ ②

PP target: $\uparrow E_t\pi_{t+1}$ for any $\downarrow \pi_t$ and thus causes $\downarrow r_t = i_t - E_t\pi_{t+1}$
$\Rightarrow$ force CB to behave symmetrically

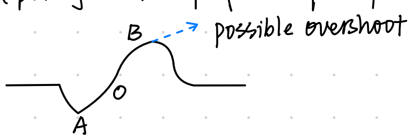
Transmission Channels

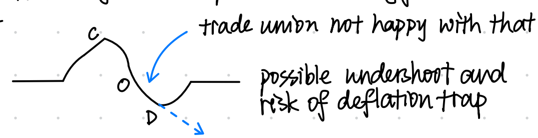
{ 1. expectations over low future nominal policy rates $\Rightarrow r_t^n \downarrow$
2. rise in expected future inflation which reduces r_t^n

① depends mainly on RE in financial sector
evidence suggests strong evidence for RE in financial markets

② depends mainly on RE in consumer / corporate sectors
because borrowers must infer existing loan offers more attractive in real terms
(some evidence consumption more heavily influenced by i than r in short term)
—— could pose questions about efficacy of PP regimes in practice

Con: control errors
inflation not perfectly controlled by CBs
(policy-makers prefer simplest possible adjustment path to PP suggests)

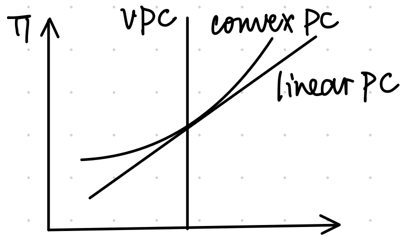




Impact of non-linearities

convex PC: obstacles to deflation

- For a given fall in output, convex PC implies smaller $\pi\downarrow$
  $\Rightarrow$ under PP regime, rise in future expected inflation limited $\Rightarrow r_t^n \uparrow$



this need not impair stabilising properties of PP regime, e.g. if decline in real rates insu¢ cient for full employment then inf would undershoot target again and this inf shortfall would further increase expected inf until demand rises sufficiently (only a completely elastic PC to LHS of VPC would nullify mechanism), and rational agents would infer this would happen and act on the implied lower real interest rates immediately

↑ require RE in consumer sector
(otherwise stabilizing properties of PP targets weaker and delayed)

Stagflation Shocks

rising import, energy, food prices $\Rightarrow \pi$ to 5%
— thus a PP for CPI would require future $\pi < 2\%$ and hence a rise in real long rates today $\Rightarrow$ exacerbate output recession

Solution: refined price measure that excludes stagflation drivers
However, coming up with a measure that works well in all circumstances is difficult.

Nominal GDP Targets

Nominal GDP = PY
1. Pure demand shocks: $P\downarrow, Y\downarrow \Rightarrow$ loose monetary policy
2. Stagflation shocks: $P\uparrow, Y\downarrow \Rightarrow$ no unnecessary tightening

However:
1. nominal GDP observed quarterly whereas CPI monthly
2. nominal GDP frequently revised due to challenges in measuring Q
3. stabilized $P = \frac{PY}{Y} = \frac{\text{nominal GDP}}{\text{real GDP}}$ (GDP deflator price concept)
   - exclude imports (consumed by households)
   - include exports (not consumed)

   $\Rightarrow$ not a good measure of cost of living than CPI

Quantitative Easing

1. entail creation of new electronic money balances for purchase of assets (such as government and corporate bonds, which boosts M^s)
2. If assets purchased at a fair price, then net worth of financial sector unchanged, but bank and investor balance sheets more liquid
   ⇒ boost lending, trigger increased expenditure to raise inflation
3. QE ⇒ lock CB into higher future inflation
   (could not quickly withdraw. a rapid sale of assets would represent an asset 'firesale' that would lower prices and impose losses on CBs)
   ⇒ a commitment technology for optimal monetary policy

Empirical Study: QE's limited effect in swaying expectations

- In 2015, inflation < 2% in US, UK and Japan
- boost the narrow money supply but not induce the bank lending needed to convert this into increased broad money supply, demand and inflation
- Monetarists: QE not aggressive enough
  (credibly commit to irresponsibility: Friedmanite helicopter drop of cash; use QE money to buy up government bonds and then cancel them)

4. QE ⇒ commercial rates = govt bond yield + risk premium(s) ↓

   in practice both the Fed and the BoJ did this through purchasing corporate bonds (loans to companies) and mortgage bonds (which fund mortgages) at prices implying lower interest rates than could be achieved in market

   (CB takes on part of the role of commercial banks through plugging a gap in the market that developed as a result of limited risk appetite)

Outlook for monetary / macro policy

1. western economies face secular stagnation
   (debt overhang, declining population growth, rising income inequality depress C and I)
2. resurgence of fiscal policy as a tool of macro stabilization
   (e.g. extensions of tax rebates for households)

Week 4:
the Open Economy

Foreign Exchange Market Model
3 equations model in open economy + AD side
- policy reaction { inflation shock
  permanent demand shock

The aggregate supply (AS) side
National Income Accounting

Foreign Exchange Market

Simplifying Assumptions:

1. Perfect international capital mobility
2. Small home country: cannot affect world interest rates
3. Households can hold 2 assets: Bonds (home / foreign), money
4. perfect substitutability between home and foreign bonds
   (only difference is expected return, same default risk)

Definitions:

Nominal exchange rate $E = \frac{\text{no. units of home currency}}{\text{one unit of foreign currency}}$

Convention: $e \equiv \ln(E)$ $e\uparrow \Rightarrow$ home currency depreciation

Real exchange rate $Q = \frac{P^* \cdot E}{P} \Leftrightarrow q = p^* + e - p$ (p^*: foreign price level)

Terms-Of-Trade $TOT \equiv \frac{1}{Q}$ a measure of relative price competitiveness

Competitiveness and Pricing Rules

- Home-cost pricing: exporters set prices based on domestic costs
- World-pricing: exporters set prices to match competitors' abroad

domestic costs $a\uparrow \Rightarrow$

- Home-cost: $P\uparrow$, $Q\downarrow$ real appreciation, loss of competitiveness
- World-pricing: $\bar{P}$, $\bar{Q}$ no apparent loss of competitiveness, margins squeezed

Better measure: relative unit labour costs (RULC)

$$RULC \equiv \frac{\text{foreign ULC in home currency}}{\text{home unit labour cost}} = \frac{ULC^* \cdot E}{ULC}$$

Three Parity Conditions

(how traders decide on home and foreign assets)

1. Purchasing Power Parity (PPP)

Consequence of arbitrage in international market for goods / services

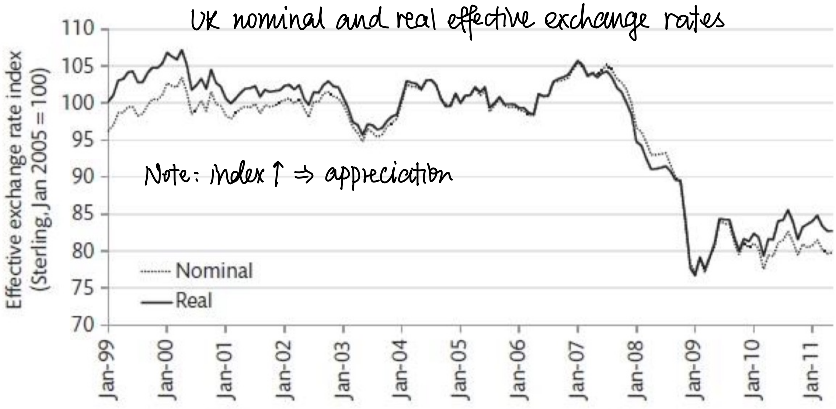
- traded goods: goods with high value-to-transportation costs, e.g., cars, crude oil
- non-traded goods: goods with low value-to-transportation costs, e.g., haircuts

never applies:
1. only traded goods
2. diff basket
3. local mkt power

Law of One Price (LOP): $P = P^* E \Leftrightarrow Q = 1$

- Absolute Purchasing Power Parity: apply to all goods; equal "baskets" of goods
- Relative PPP: $\Delta\% Q = 0 \Leftrightarrow \Delta\% E = \Delta\% P - \Delta\% P^* = \pi - \pi^*$
  allows for fundamental differences in prices due to barriers such as tariffs

However, little evidence of even relative PPP:



2. Covered Interest Parity (CIP)

Arbitrage in FX market: any interest rate differential must be compensated by the forward premium (or discount):

$$i_t - i_t^* = \frac{F_{t+1} - E_t}{E_t} \quad (F = \text{forward rate},\ E = \text{spot rate})$$

If $i_t - i_t^* > 0 \Rightarrow F_{t+1} > E_t$ (home currency depreciates in 1 year's time)

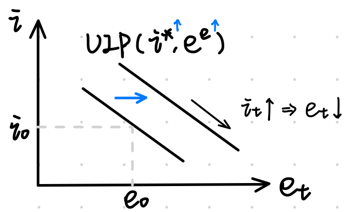
3. Uncovered Interest Parity (UIP)

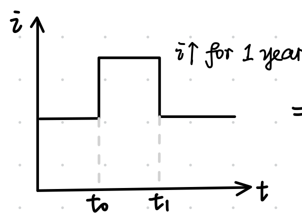
Similar to CIP, except that not covered with a forward contract:

$$i_t - i_t^* = \frac{E^e_{t+1} - E_t}{E_t} \quad (E^e_{t+1} \equiv E_t(E_{t+1})) = e^e_{t+1} - e_t$$

More reflection of speculation on future evolution of exchange rate than arbitrage.

Given e^e_{t+1}, $e_t = f(i_t, i_t^*)$, $(-, +)$





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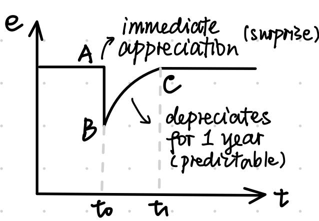


Interest gain from holding H rather than F bonds
= Loss from expected H currency depreciation against F's

3 Equations Model in Open Economy

Assumptions:

1. Small open economy: r ultimately fixed abroad by r^* (world interest rate) rather than by domestic markets (short run deviation allowed)
2. Exchange rate regime
   - Fixed: e pegged to a foreign currency ⇒ no autonomy of policy
   - Floating: e determined by UIP, consistent with inflation target
3. Home's inflation target $\pi^T = \pi^*$ (world inflation)

previous: $y_t - y_e = -a(r_{t-1} - r_s)$ ⇒ "A"

IS curve: $y_{t+1} = A - ar_t + bq_t$ (Marshall-Lerner: $NX^{\uparrow\uparrow} = X^{\uparrow} - M^{\downarrow}$ increases with q_t)

Phillips curve: $\pi_{t+1} = \pi_t + \alpha(y_{t+1} - y_e)$ (WLOG firms: home cost pricing)

Monetary rule: $(y_{t+1} - y_e) = -\alpha\beta(\pi_{t+1} - \pi^T)$

A fourth equation: real UIP

simplify notation: $i - i^* = e^e_{t+1} - e_t = E(\Delta e)$

subtract $E(\pi - \pi^*)$ from both sides: $i - (\pi - \pi^*) = i^* + E(\Delta e) - (\pi - \pi^*)$

$\Rightarrow r = r^* + E(\Delta e) - E(\pi - \pi^*)$

from $Q = \frac{P^*E}{P} \Leftrightarrow q = p^* + e - p$ and $\Delta q = E(\Delta p^*) + \Delta e - E(\Delta p)$, thus $E(\Delta q) = E(\Delta e) - E(\pi - \pi^*)$

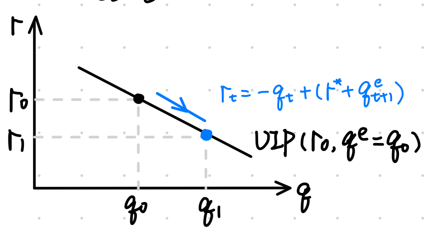
$\Rightarrow r - r^* = q^e_{t+1} - q_t = E(\Delta q)$ (real)

Also as: $q_t = -(r_t - r^*_t) + E_t(q_{t+1})$ Also: $e_t = E_t(e_{t+\infty}) + E_t \sum_{i=0}^{\infty} (i^*_{t+i} - i_{t+i})$ (nominal)

repeated forward substitution: $q_t = -E_t \sum_{i=0}^{\infty} (r_{t+i} - r^*_{t+i}) + E_t[q^{\text{Long Run}}]$

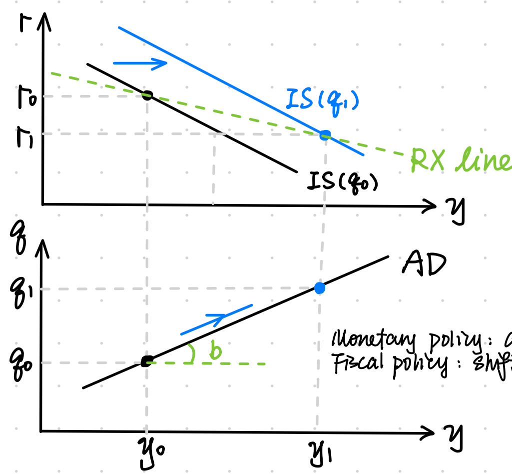
(today's exchange rate = long run value + sum of all current and future interest differences

1. The Aggregate Demand (AD) side



Economy starts in short-run equilibrium: (r_0, q_0, y_0)

- CB cuts interest rate to r_1 ↓
  ⇒ ↑ domestic AD to y'
  ↑ e to clear FX market (UIP)
  ↑ real depreciation (q↑)
  foreign demand pushes income up to y_1 (IS shifts)



Monetary policy: along AD
Fiscal policy: shifts of AD

AD curve ≡ short-run equilibria (labour mkt not necessarily)

2. Policy Reactions

(1) Inflation shock: Inflation jumps above targets (e.g. oil shock)

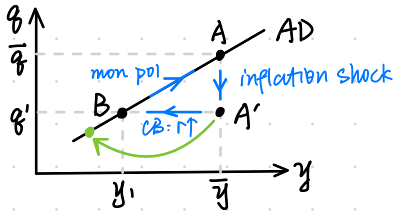
News of $\pi\uparrow$ ⇒ Immediate real appreciation ($q\downarrow$) because of price rigidity

⇒ CB fights it by: $r\uparrow$

⇒ { negative output gap $y\downarrow$
  | $q\downarrow$ (appreciates) ⇒ $y\downarrow$ ($EX\uparrow$, $AD\uparrow$) returns to Home bonds $\uparrow$

⇒ $\pi\downarrow$, CB eases r as returning to equilibrium

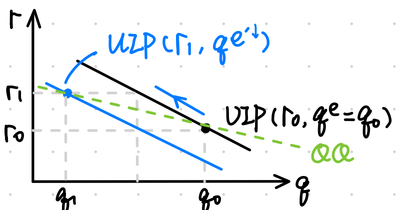
(Interest rate channel + Exchange Rate channel)

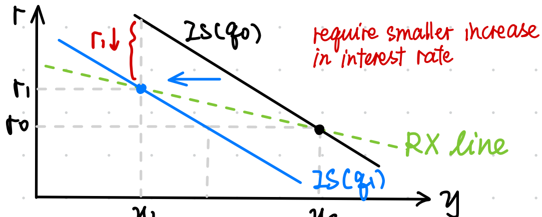


In an open economy:
$r\uparrow \Rightarrow q\downarrow \Rightarrow (X-M)\downarrow \Rightarrow Y\downarrow$
Hence the required Δr smaller

$r_t^{\uparrow} - r^* = E_t q_{t+1}^{\downarrow} - q_t^{\downarrow\downarrow}$ (Because expect q_t to fall ⇒ $q^e\downarrow$)

FX market: expect $\pi\downarrow$ ⇒ expectations of appreciation ⇒ UIP shifted left





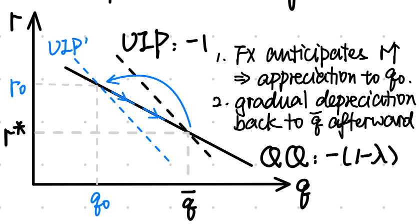
initial appreciation ←

subsequent depreciation →

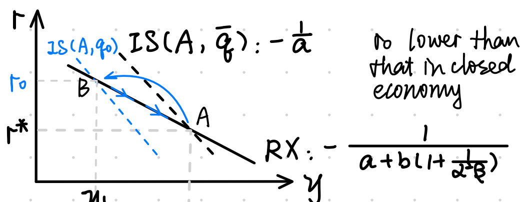
$q^{e'} < q^e = q_0$

CB cuts r ⇒ UIP shifts back as $q^{e'}\uparrow$

Small Open Economy Model



1. FX anticipates $r\uparrow$ ⇒ appreciation to q_0.
2. gradual depreciation back to $\bar{q}$ afterward



no lower than that in closed economy

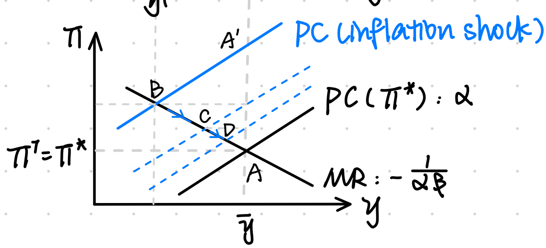
$UIP:\ r = -q_t + q^e_{t+1} + r^*$

$IS:\ y_{t+1} = A - ar_t + bq_t$

$PC:\ \pi_{t+1} = \pi_t + \alpha(y_{t+1} - y_e)$

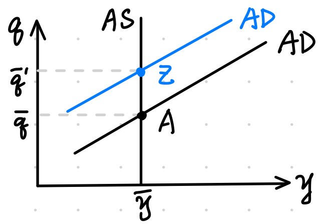
$MR:\ y_{t+1} - y_e = -\alpha\beta(\pi_{t+1} - \pi^T)$

All initial appreciation subsequently reversed

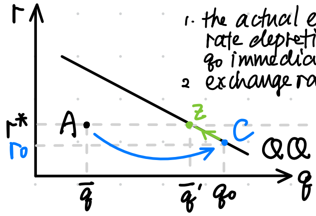


(2) **Permanent demand shock**: change in world demand for domestic exports

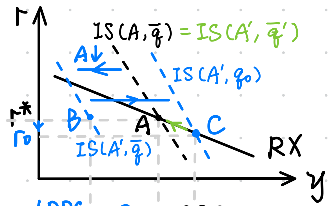
simplifying assumption: vertical AS (same $\bar{y}$), depreciated currency ($\bar{q}' > \bar{q}$)

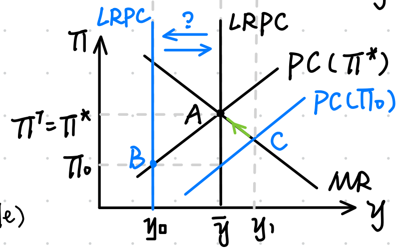


$$UIP: r = -q_t + q^e_{t+1} + r^*$$
$$IS: y_{t+1} = A - ar_t + bq_t$$
$$PC: \pi_{t+1} = \pi_t + \alpha(y_{t+1} - y_e)$$
$$MR: y_{t+1} - y_e = -\alpha\beta(\pi_{t+1} - \pi^T)$$



1. the actual exchange rate depreciates to q_0 immediately
2. exchange rate overshoots





Part of initial depreciation subsequently reversed.

—— Dornbusch (1976) called this 'overshooting'

(explain why exchange rates so volatile)

Exchange rates are influenced by interest rates and expectations, which may change rapidly, making exchange rates volatile.

Assumptions for overshooting

1. Uncovered Interest Parity
2. Short-run rigidity of the price (and wage) level
3. rational expectations
   - about inflation
   - about exchange rates q^e
4. Expectations about inflation adjust quickly, but prices adjust only in the medium run

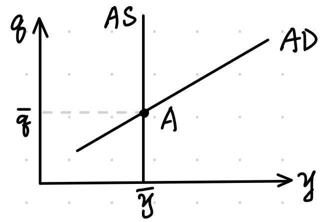
   Exchange rate must overshoot initially (in real terms) to clear FX market

Application: Exchange Rate overshooting in the UK

Tightening of MP (1979) did not lead to an immediate fall in π, but to a sharp and long-lasting appreciation in the pound.

The Aggregate Supply (AS) side

So far have been assuming a vertical AS curve (requires labour market to be indifferent to q)



Unlikely:
- imported component of consumption affects real consumption wage
- imported inputs affects firms' production costs

Home currency appreciation ⇒ cheaper IM
⇒ workers happier, firms neutral (ease distribution tension)

Labour market in open economy

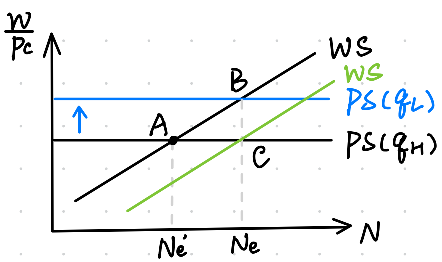
With imperfect competition (market power): $P = (1+\underbrace{\mu}_{\text{mark-up}})\dfrac{W}{\underbrace{\lambda}_{\text{MPL}}}$

consumer price index: $P_c = (1-\phi)P + \underbrace{\phi}_{\text{share of imported goods}} P^* E$

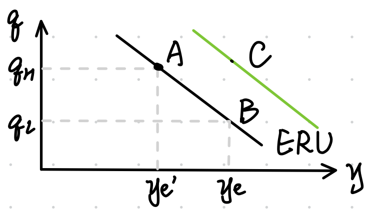
Derivation of price-setting (PS) real wage equation:

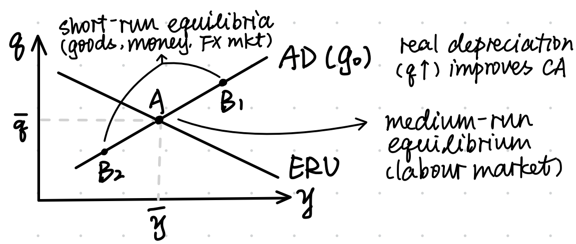
$$\Rightarrow \frac{W}{P_c} \equiv w^{PS} = \frac{\lambda(1-\mu)}{1+\phi(Q-1)}$$

$q\downarrow$ (real appreciation) ⇒ w^{PS} shifts up



ERU and q.
(Equilibrium Rate of Unemployment)
- Negative relation between q and w^{PS}
- Structural reforms in labour market shift the whole ERU





Along ERU: stable price and wage inflation
Underlying assumption:
1. Country can finance any deficit of its CA
2. Stable exchange rate expectations in the middle run

National Income Accounting

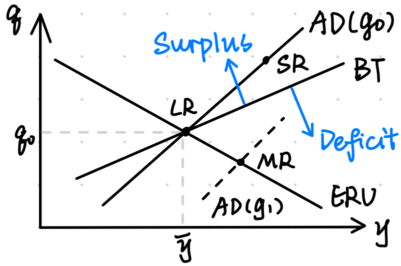
National Expenditure: $Y \equiv C + I + G + (X - M)$

Current Account balance: $CA = X - M = BT = \underbrace{(S-I)}_{\text{private balance}} + \underbrace{(T-G)}_{\text{public balance}}$

Balance of payments: $CA + \underbrace{FA}_{} = 0$

Financial Account: the outflow of net financial assets

'Long run' Equilibrium



Balanced Trade (BT) line

BT positive slope:

if $q\uparrow$ (depreciation) $\Rightarrow X-M\uparrow$ (Marshall-Lerner)

$\Rightarrow$ need $M\uparrow$ (via y) to keep $BT=0$

BT slope lower than AD:

$q\uparrow \Rightarrow X-M\uparrow$, part of the income$\uparrow$ will be saved (or taxed), so goods market eqm occurs at lower level of y.

1. At SR, $BT>0 \Rightarrow q^e<0$ (appreciation)
   $X-M\downarrow$, $y\downarrow$, shift down AD till LR
2. At MR, $BT<0 \Rightarrow q^e>0$ (depreciation)
   But labour market puts pressure for appreciation $\Rightarrow$ unstable

Application: the UK economy 1997-2008

Week 5:
Growth Model

Exogenous and Endogenous Growth Models
- the Solow-Swan Model
- the Romer / Jones Model

Income Inequality: the Galor-Moav Model

Empirical Evidence
- Convergence, Middle Income Trap
- Growth Accounting
- Income Inequality
- The Golden Rule
- R&D investment
- Fertility

Exogenous and Endogenous Growth

Mathematical Tools:

$X_{t+1} - X_t$: the change in X between today and tomorrow

$\dot{X}_t := \frac{dX_t}{dt}$: the change in X between now and instant immediately after

Definitions of growth rates.

in discrete time $g_X := \frac{X_{t+1} - X_t}{X_t} \approx \log(X_{t+1}) - \log(X_t)$

in continuous time $g_X := \frac{\dot{X}_t}{X_t} = \frac{d\log(X_t)}{dt}$

The Solow Model (1956, 1957)

Growth exogenously comes from technology and/or population

$$\begin{cases} \text{production function: } Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \\ \text{law of motion of capital: } \dot{K}_t = I_t - \delta K_t \\ \text{investment rule: } I_t = sY_t \\ \text{resource constraint: } Y_t = C_t + I_t \\ \text{population growth: } \dot{L}_t = g_L L_t \\ \text{technology growth: } \dot{A}_t = g_A A_t \end{cases}$$

1. Production Function: $Y_t = K_t^{\alpha}(\underline{A_t L_t})^{1-\alpha}$ time-variant

GDP is produced with three inputs: capital, labour, labour-augmenting tech

Empirical estimates: $\alpha \sim (0.3, 0.4)$

$F: \mathbb{R}_+^3 \to \mathbb{R}_+$ s.t. $F: (K_t, A_t, L_t) \longmapsto Y_t$

Non-negativity: $F(K_t, A_t L_t) \geq 0$, $\forall (K_t, A_t, L_t) \in \mathbb{R}_+^3$

Monotonicity: $\frac{\partial F}{\partial X_t} > 0$. $\forall X_t \in \{K_t, A_t, L_t\}$

Concavity: $\frac{\partial^2 F}{\partial X_t^2} < 0$. $\forall X_t \in \{K_t, A_t, L_t\}$

Inada conditions: $\lim_{X_t \to 0} \frac{\partial F}{\partial X_t} = \infty$, $\lim_{X_t \to \infty} \frac{\partial F}{\partial X_t} = 0$. $\forall X_t \in \{K_t, A_t, L_t\}$

Constant returns to scale (CRS): $F(\lambda K_t, \lambda A_t L_t) = \lambda Y_t$, $\forall \lambda \in \mathbb{R}_+$

Types of Technology:

$\left.\begin{array}{l} \text{1. Hicks-neutral } Y = AF(K, L)\text{, } A\text{: Total Factor Productivity} \\ \text{2. Harrod-neutral } Y = F(K, AL)\text{. } A\text{: Labour-augmenting} \\ \text{3. Solow-neutral } Y = F(AK, L)\text{. } A\text{: Capital-augmenting} \end{array}\right\}$ Cobb-Douglas production function

Technological progress (i.e., $A\uparrow$) is said to be directed or input-biased if not Hicks-neutral

2. The Law of Motion of Capital: $\dot{K}_t = I_t - \delta K_t$

I_t: active investment, δK_t: passive depreciation, $\delta \in (0,1)$, estimated 3% yearly

3. Resource Allocation: $I_t = sY_t$, $Y_t = C_t + I_t$

s: marginal propensity to save (not the average saving rates, assumed to coincide)

Microfoundation:

(Household) $\max_{C_t, C_{t+1}, I_t} \log(C_t) + \beta \log(C_{t+1})$, s.t. $C_t + I_t = Y_t$, $C_{t+1} = I_t$, Y_t, β given

$\Rightarrow I_t^* = \frac{\beta}{1+\beta} Y_t = sY_t$

The more households discount the future ($\beta\downarrow$) $\Rightarrow$ the less they save ($s\downarrow$)

4. Sources of Growth: $\dot{L}_t = g_L L_t$, $\dot{A}_t = g_A A_t$

Exogenous and constant net growth rate g_L, g_A
could be affected by policy (e.g., fertility policies ; R&D subsidies)

Steady Growth:

capital per effective unit of labour: $k_t = \frac{K_t}{A_t L_t}$

capital depth: K_t / L_t (how much capital each worker uses in production)

definition of steady state: $\dot{k}_t = 0$

(achieving time-invariant growth rates in the steady state)

Intensive Form:

Define $x_t := \frac{X_t}{A_t L_t}$ for each $X_t \in \{Y_t, I_t, C_t, K_t\}$:

production function: $y_t = k_t^{\alpha}$ (derived from $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$)

law of motion: $\dot{k}_t = i_t - (g_A + g_L + \delta) k_t$

$$\dot{k}_t := \frac{d}{dt}\left(\frac{K_t}{A_t L_t}\right) = \frac{\dot{K}_t A_t L_t - K_t \dot{A}_t L_t - K_t A_t \dot{L}_t}{A_t^2 L_t^2} = \frac{\dot{K}_t}{A_t L_t} - \frac{K_t}{A_t L_t} \cdot \left(\frac{\dot{A}_t}{A_t} + \frac{\dot{L}_t}{L_t}\right)$$

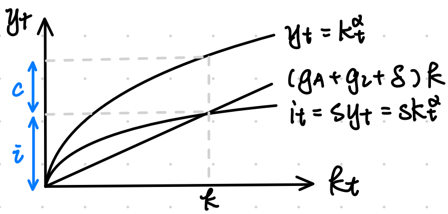
$$= \frac{I_t - \delta K_t}{A_t L_t} - (g_A + g_L) k_t = i_t - (g_A + g_L + \delta) k_t$$

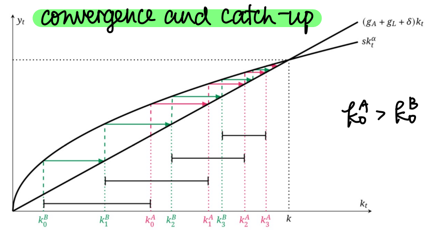
$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - g_A - g_L := g_K - g_A - g_L$$

α: capital share of income $\Rightarrow$ $sk^{\alpha} = (g_A + g_L + \delta) k$ (investment = "depreciation") (from $i_t = sy_t = sk_t^{\alpha}$)

$\Rightarrow$ the curvature of the production function $\Rightarrow$ $k = \left(\frac{s}{g_A + g_L + \delta}\right)^{\frac{1}{1-\alpha}}$

$\Rightarrow \alpha \uparrow \Rightarrow MPK \uparrow$





$k_0^A > k_0^B$

Growth and Balanced-Growth Path (BGP)

Balanced growth: all variables grow at constant rates

—— Kaldor's Fact (1961):

1. Y_t / L_t exhibits continual growth
2. K_t / L_t exhibits continual growth
3. K_t / Y_t is roughly constant over time

Thus, GDP grows according to:

$$g_Y = \frac{d \log(Y_t)}{dt} = \alpha g_K + (1-\alpha)(g_A + g_L)$$

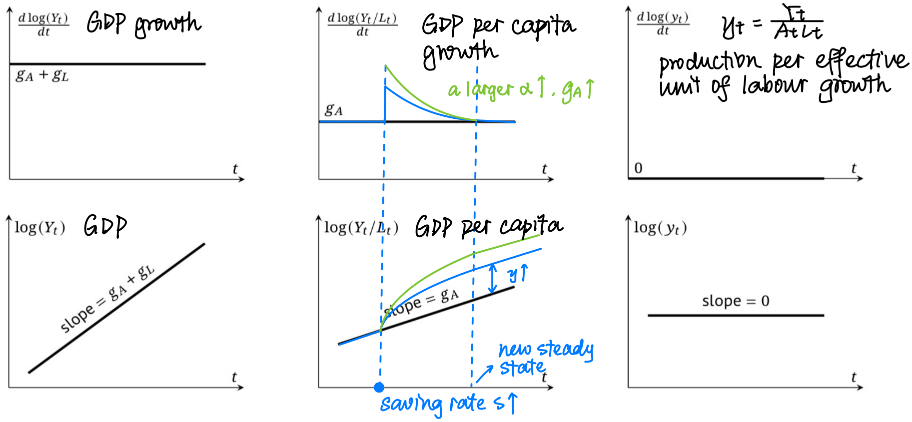
proof: $g_Y = \frac{d}{dt}\left(\log\left(K_t^{\alpha}(A_t L_t)^{1-\alpha}\right)\right) = \frac{d}{dt}\left(\alpha \log(K_t) + (1-\alpha)\log(A_t L_t)\right)$

$$= \alpha \frac{d \log(K_t)}{dt} + (1-\alpha)\left[\frac{d\log(A_t)}{dt} + \frac{d\log(L_t)}{dt}\right] = \alpha g_K + (1-\alpha)[g_A + g_L]$$

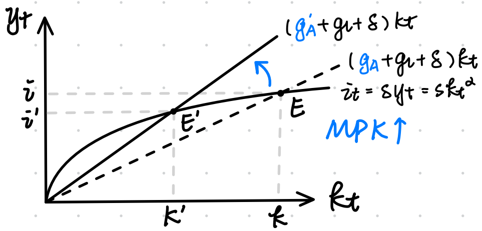
Definition for BGP: $g_Y = g_K \Rightarrow g_Y^{BGP} = g_A + g_L$

Growth along the BGP in the steady state:

In the steady state, the model predicts *constant* (*i*) GDP growth and (*ii*) GDP per capita growth

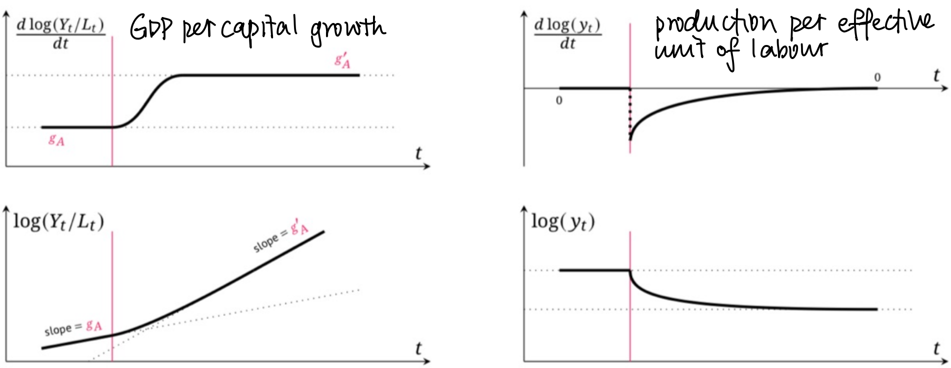


Technological Progress:



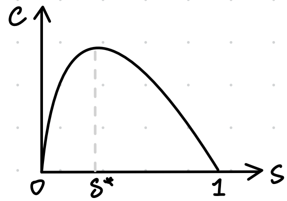
⇒ there is less capital per 'effective worker' (diminishing MPK)
(i.e., capital thinning)

⇒ tech increases 'effective workers', as tech is labour augmenting
Dominates : linear !



The Golden Rule

$$c = (1-s)y = (1-s)\left(\frac{s}{g_A+g_L+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$



Key Equations: $y = c + i$, $y = k^\alpha$

↑: MPK high, I↑ ⇒ C↑, 'size-of-cake' effect

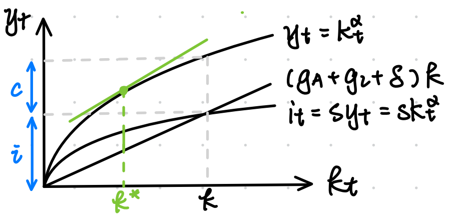
↓: MPK low, I takes sources away from C

'share-of-cake' effect

Abusing the resource constraint: $y_t = (1-s)y_t + sy_t$

⇒ Golden Rule: $\max_s (1-s)\left(\frac{s}{g_A+g_L+\delta}\right)^{\frac{\alpha}{1-\alpha}}$

FOC: $MPK = g_A + g_L + \delta$



steady state consumption:

$$c = (1-s)y = (1-s)(AL)^\alpha\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

A↑ (improvement in technology) ⇒ c↑

$\frac{\partial c}{\partial A} > 0$, $\frac{\partial^2 c}{\partial A \partial \delta} < 0$, higher depreciation ⇒ $\frac{\partial A}{\partial c}$↓

The Romer / Jones Model (1990, 1999)

Romer: technological growth might depend on population growth

—— people produce ideas based on existing ideas (endogenize tech progress)

$$\dot{A}_t = \nu A_t^\phi L_t^A$$

- $\phi < 0$: decreasing RTS (fishing out ideas)
- $\phi = 0$: constant RTS
- $\phi > 0$: increasing RTS (standing on giants' shoulders)

Romer: $\phi = 1$, Jones: $\phi < 1$

- production function: $Y_t = K_t^\alpha (A_t L_t^Y)^{1-\alpha}$
- law of motion of capital: $\dot{K}_t = I_t - \delta K_t$
- investment rule: $I_t = sY_t$
- resource constraint: $Y_t = C_t + I_t$
- labour resources: $L_t = L_t^A + L_t^Y$
- labour allocation: $L_t^A = \ell L_t$, ℓ: proportion of labour to R&D
- population growth: $\dot{L}_t = g_L L_t$
- R&D production: $\dot{A}_t = \nu A_t^\phi L_t^A$

⇒ In a BGP, output growth is: $g_Y^{BGP} = g_A + g_L = \gamma A_t^{\phi-1} L_t^A + g_L$

Romer ($\phi=1$). $g_Y^{BGP} = \gamma L_t^A + g_L$

1. GDP growth depends on both population growth and population level
2. Scale effect due to IRS in production of ideas
3. BGP features exponential (explosive) growth
4. Policy take-away: control growth by directing more or less labour to R&D (i.e., ℓ)

Jone's (1999) critique

$g_A = \gamma A_t^{\phi-1} L_t^A$ non-explosive $\Rightarrow \dot{g}_A = 0$

$\frac{dg_A}{dt} = 0 \Rightarrow \frac{d}{dt}\frac{\dot{A}_t}{A_t} = 0 \Rightarrow \frac{d}{dt}(\gamma A_t^{\phi-1} \cdot \ell L_t) = 0$

$\Rightarrow (\phi-1) A_t^{\phi-2} \dot{A}_t \cdot L_t + A_t^{\phi-1} \dot{L}_t = 0 \Rightarrow (1-\phi) g_A \cdot A_t \cdot L_t = A_t \cdot g_L L_t \Rightarrow g_A = \frac{g_L}{1-\phi}$

$\Rightarrow g_A = \frac{g_L}{1-\phi}$, thus $g_Y^{BGP} = \frac{2-\phi}{1-\phi} g_L$, with $\phi < 1$

IRS in R&D need not imply scale effects, squaring with evidence

Microfoundations: Increasing RTS and Competition

The models in P. M. Romer (1990) and Jones (1999) feature IRS in R&D production

- Incompatible with the idea of perfect competition
- Say we have many atomistic firms producing with some IRS technology
- IRS implies $MC < AC$, regardless of the final quantity produced
- Perfect competition implies $P = MC$
- Hence $P < AC$ implies negative profits: firms will leave the market
- Only one firm survives and operates as a monopolist

P. M. Romer (1990) addresses this transparently with a full model and a “trick”

- 3 markets: final good, intermediate goods, R&D
- Final goods: firms in perfect competition
- Intermediate goods: one monopolist with a *patent*
- R&D: firms in perfect competition (e.g., free entry)

We do **not** get into the weeds

- Analyses at aggregate level
- How agents’ actions aggregate up is left out of the picture

Growth and Income Inequality

The Galor-Moav Model (2000):

$\begin{cases} \text{in the long run, growth is just like in Solow} \\ \text{in the short run, growth exacerbates income inequality} \end{cases}$

Key ingredients: 1. skilled-biased technological change
2. workers with heterogeneous ability
3. occupational choice: skilled and unskilled workers

Setup: Labour Force

$$H_t := \beta \underbrace{h_t}_{\text{skilled labour}} + (1-\delta g_t) \underbrace{l_t}_{\text{unskilled labour}} \quad \text{, where } g_t = \frac{\dot{A}_t}{A_t} = \frac{A_t - A_{t-1}}{A_{t-1}}$$

1. technological progress is skill-biased
the growth rate of technology $g_t \uparrow \Rightarrow$ demand for unskilled $\downarrow$
2. in the long run, g_t is constant and so ratio l_t / h_t unaffected

Setup: Labour Demand

Firms maximize profits in perfect competition:

$$\max_{Y_t, K_t, H_t} Y_t - r_t K_t - w_t H_t \quad \text{s.t. } Y_t = K_t^{\alpha} (A_t H_t)^{1-\alpha}$$

$$\Rightarrow \text{FOCs}: \; r_t = \alpha k_t^{\alpha-1}, \; w_t = A_t (1-\alpha) k_t^{\alpha} := A_t \mu_t \quad (\mu_t = (1-\alpha) k_t^{\alpha})$$

Simplifying assumptions:
1. small open economy without perfect capital mobility
$\Rightarrow r_t = r$ fixed, $k_t = k$ fixed $\Rightarrow \mu_t = \mu$ fixed
2. the composite wage is allocated to (un)skilled wages according to efficiency weights:

$$\begin{cases} w_t^u = (1-\delta g_t) A_t \mu \\ w_t^s = \beta A_t \mu \end{cases}$$

(*) Remark: skill premium $\omega := \frac{w_t^s}{w_t^u} = \frac{\beta}{1-\delta g_t}$

Setup: Labour Supply

Assumptions:
1. Workers are heterogeneous because of ability $a_t^i \sim U[0,1]$
But each worker has one unit of time.

2. Workers decide whether to get education at time cost $\tau \in (0,1)$. Workers with(out) education are (un)skilled.

(*) Remark on terminology and concept:

1. Ability: innate, non-transferable, unobservable, affects hours worked
2. Skill: acquired, transferable, observable

Labour supply (in efficiency units)

1. unskilled labour: $l_t^i = 1-(1-a_t^i)g_t$
   - unskilled workers work all the time
   - when technology changes, some time is dedicated to learning the new tools
   - time spent learning new tech is lower for more able workers

unskilled labour income in equilibrium:

$$I_t^{i,u} = W_t^u l_t^i = (1-\delta g_t)A_t\mu \cdot [1-(1-a_t^i)g_t]$$

2. skilled labour: $h_t^i = (1-\tau)[a_t^i-(1-a_t^i)g_t]$
   - skilled workers work after having spent time fraction τ acquiring skills
   - ability has a positive effect on the efficiency of labour units supplied
   - when technology changes, some time is dedicated to learning the new tools
   - time spent learning new tech is lower for more able workers

skilled labour income in equilibrium:

$$I_t^{i,s} = W_t^s \cdot h_t^i = \beta A_t\mu \cdot (1-\tau)[a_t^i-(1-a_t^i)g_t]$$

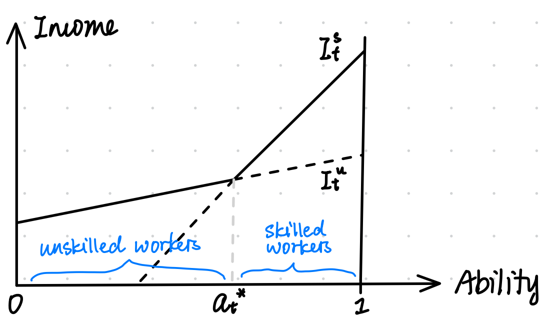
Workers decide whether to acquire skills comparing incomes:

$I_t^{i,s}(a_t^i, A_t, g_t) \geq I_t^{i,u}(a_t^i, A_t, g_t)$

$\Rightarrow \beta A_t\mu \cdot (1-\tau)[a_t^i-(1-a_t^i)g_t] \geq (1-\delta g_t)A_t\mu \cdot [1-(1-a_t^i)g_t]$

$\Rightarrow a_t^i \geq a_t^* = \frac{1-\delta g_t+\delta g_t^2}{1+\delta g_t^2}$ (simplifying assumption: $\beta(1-\tau)=1$)

threshold ability level depends on tech growth g_t, not on tech level A_t



Steady State: $g_t = g$ constant

Important to distinguish:

1. Changes in tech level A_t (always present, i.e., short / long run)
2. Changes in tech progress g_t (only present away from steady state, i.e., short-run)

Effects of Growth on Labour Income

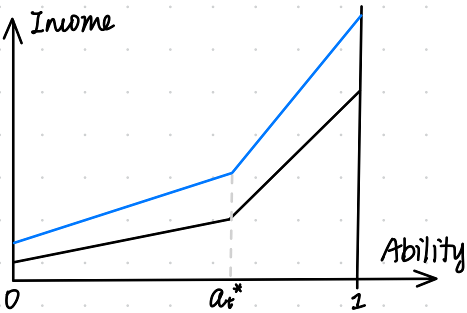
Three channels:

1. productivity effect
2. erosion effect
3. composition effect

$$I_t^{u,avg} = \frac{A_t \mu (1-\delta g_t)[2-g_t+(1-g_t)\delta g_t^2]}{2(1+\delta g_t^2)}$$

$$I_t^{s,avg} = \frac{A_t \mu (2-\delta g_t+\delta g_t^2)}{2(1+\delta g_t^2)}$$

1. Productivity Effect (Long Run)

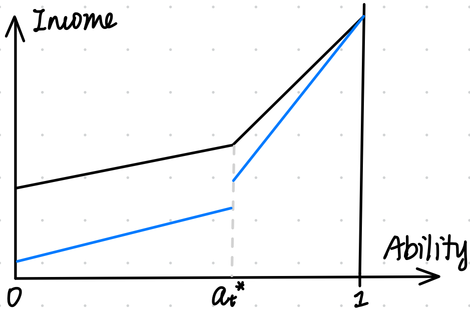


An increase in $A_t \uparrow$, $g_t = g$ constant:

- increase everyone's productivity
- increase wages (and so income) for everyone, but less for unskilled

$$\begin{cases} \dfrac{\partial I_t^{i,u}}{\partial A_t} = (1-\delta g_t)\mu[1-(1-a_t^i)g] > 0 \\ \dfrac{\partial I_t^{i,s}}{\partial A_t} = \beta\mu(1-\tau)[a_t^i-(1-a_t^i)g] > 0 \end{cases}$$

2. Erosion Effect (Short Run)



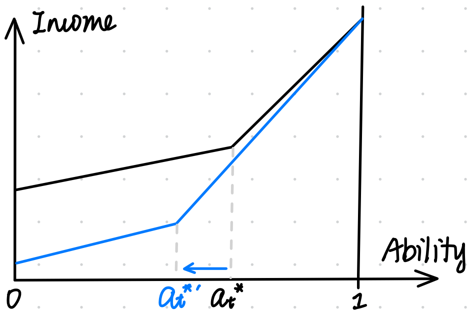
An increase in $g_t \uparrow$, a_t^* fixed (!):

- decrease I^s only through h
- decrease I^u through w^u, ℓ

wage premium $\omega_t = w_t^s / w_t^u \uparrow$

$$\begin{cases} \dfrac{\partial I_t^{u,avg}}{\partial A_t} = -\dfrac{A_t\mu}{2}[\delta(2-(2-a_t^*)g_t)+(1-\delta g_t)(2-a_t^*)] < 0 \\ \dfrac{\partial I_t^{s,avg}}{\partial A_t} = \dfrac{A_t\mu}{2}[a_t^*-1] < 0 \end{cases}$$

3. Composition Effect (Short Run)

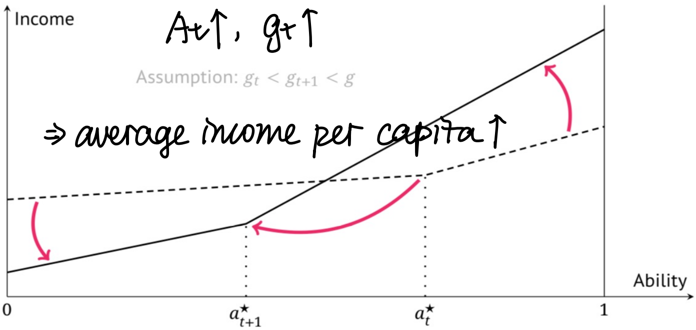


An increase in $g_t \uparrow$:

- increase wage premium
- $\Rightarrow$ unskilled workers re-evaluate their occupational choice
- $\Rightarrow$ decrease $\ell \downarrow$ (avg. ability $\downarrow$)
- $\Rightarrow$ increase $h \uparrow$ (avg. ability $\downarrow$)

$$\frac{\partial a_t^*}{\partial g_t} = -\frac{\delta[(1-\delta g_t^2)+g_t^2(1-\delta)]}{(1+\delta g_t^2)^2} < 0$$

From t to t+1: Productivity + Erosion + Composition



From period t to period $t+1$, away from the steady state, A_t increases and so does g_t

Productivity	visible for $a_t^i = 1$ (no erosion effect for most able worker)
Erosion	visible for $a_t^i = 0$ (minimal productivity effect for least able worker)
Composition	visible for moving occupational choice threshold from $a_t^\star$ to $a_{t+1}^\star$

Income Inequality

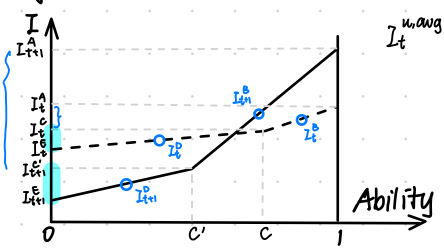
Five reference workers to analyse inequality

Name	Description	Ability a_t^i	Income
Alice	Most able	1	$I(1, A_t, g_t)$
Bob	Average skilled	$(a_t^\star + 1)/2$	$I_t^{s,\text{avg}}$ ▸ Maths
Charlie	Marginal	$a_t^\star$	$I(a_t^\star, A_t, g_t)$
Diana	Average unskilled	$a_t^\star/2$	$I_t^{u,\text{avg}}$ ▸ Maths
Elon	Least able	0	$I(0, A_t, g_t)$

Three measures of income inequality

Inequality	Incomes of	Symbol
Within unskilled workers	$I_t^{\text{Charlie}}/I_t^{\text{Elon}}$	σ_t^u
Within skilled workers	$I_t^{\text{Alice}}/I_t^{\text{Charlie}}$	σ_t^s
Between groups	$I_t^{\text{Bob}}/I_t^{\text{Denise}}$	$\sigma_t^{s/u}$

Effect of growth on income inequality:



$$I_t^{u,avg} = \frac{A_t\mu(1-\delta g_t)[2-g_t+(1-g_t)\delta g_t^2]}{2(1+\delta g_t^2)}, \quad I_t^{s.avg} = \frac{A_t\mu(2-\delta g_t+\delta g_t^2)}{2(1+\delta g_t^2)}$$

$$\nabla_t^u = \frac{1}{(1-g_t)(1+\delta g_t^2)}, \quad \frac{\partial \nabla_t^u}{\partial g_t} > 0 \quad \uparrow$$

$$\nabla_t^s = \frac{1+\delta g_t^2}{1-\delta g_t^2}, \quad \frac{\partial \nabla_t^s}{\partial g_t} > 0 \quad \uparrow$$

$$\nabla_t^{s/u} = \frac{(2-\delta g_t+\delta g_t^2)}{(1-\delta g_t)[2-g_t+(1-g_t)\delta g_t^2]} \quad \uparrow$$

Empirical Evidence

Convergence

Solow model predicts convergence:

If two countries share similar parameters, they will eventually achieve similar growth

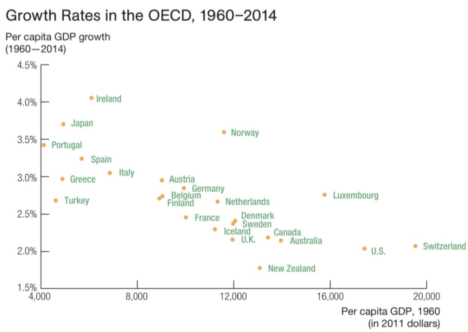
Types of convergence dynamics:

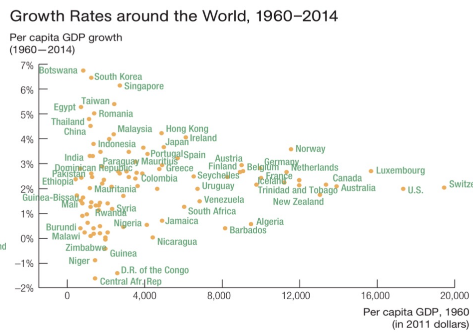
- Absolute (unconditional): countries with lower Y_t/L_t grow faster
- Conditional: … depending on the structural characteristics
- Club: … depending on their initial conditions

Types of convergence measures:

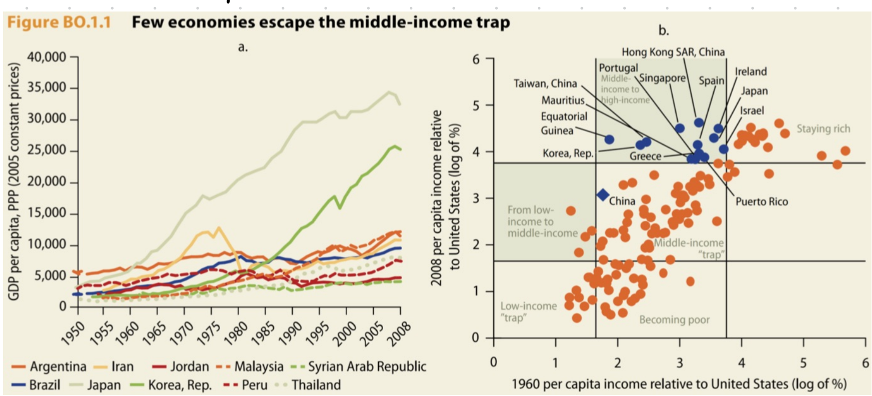
$\begin{cases} \beta: \text{income per capita (levels)} \\ \sigma: \text{income inequality} \end{cases}$

(*) β convergence $\Rightarrow$ σ convergence, but not the other way around





Middle Income Trap



Growth Accounting

Under Cobb-Douglas production function:

$$Y_t = \underbrace{A_t}_{\text{Total Factor Productivity (a.k.a., Solow residual)}} K_t^{\alpha} L_t^{1-\alpha} \Rightarrow g_{Y/L} = g_A + \alpha g_{K/L}$$

Empirical study: $g_{Y/L}, g_{K/L}$ observed, α estimated $\Rightarrow g_A$ can be computed

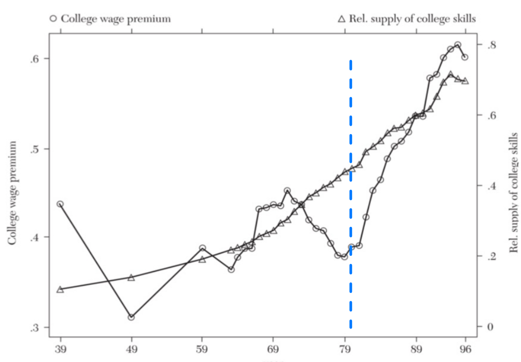
Imome Inequality

Critique for Galor and Moav (2000):

income inequality even exists in the far past

Acemoglu (2002) tackles these challenges:

1. In last 60 years, relative demand for skills shift $\Rightarrow$ skill biased
2. After 1970s,
   steady demand: relative supply of skills has not kept up
   accelerating skill bias: skill-biased tech change sped up



Relative Supply of College Skills and College Premium

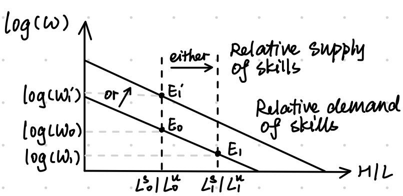
1. Galor-Moav cannot explain 1939-1979
2. after 1980, skill premium skyrocketed

Firm: $\max_{\{Y,L,H\}} Y - W^u L^u - W^s L^s$ s.t. $Y = \left[(A^u L^u)^{\frac{\sigma-1}{\sigma}} (A^s L^s)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$

FOC (perfectly competitive labour market): $\begin{cases} W^u = MPL^u \\ W^s = MPL^s \end{cases}$

the log skill premium: $\log(\omega) = \log\left(\frac{W^s}{W^u}\right) = \frac{\sigma-1}{\sigma}\log\left(\frac{A^s}{A^u}\right) - \frac{1}{\sigma}\log\left(\frac{L^s}{L^u}\right)$

$\Rightarrow$ if $\sigma > 1$, unskilled and skilled labour are (imperfect) substites (empirically plausible)



Outward demand shifts occur when A^s increases relative to A^u, but only if $\sigma > 1$ (labour types are imperfect substitutes) —— for the Galor-Moav to fit the data

The Golden Rule

Challenge for Solow: all savings eventually directed to productive investment?

Total debt = $\underbrace{\text{Household debt + Corporate debt}}_{\text{private debt}}$ + Public debt

⇒ might be used to anticipate consumption (e.g., credit card)

⇒ might be used to finance public consumption (e.g., NHS)

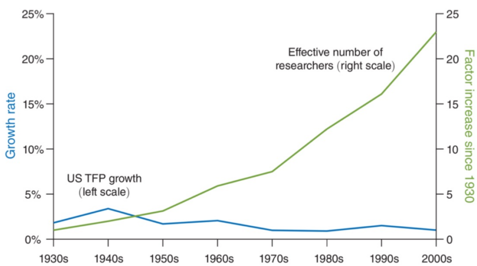
- Should a country pursue the Golden Rule before reaching steady growth?
  - Soviet Union: high saving rate, but low current consumption and low K_t/L_t
  - if population is large, high saving rate might make people hungry

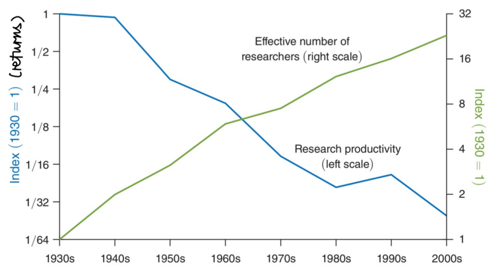
R&D Investment

Romer and Jones: ideas are produced with increasing RTS

Critiques: 1. Bloom, Jones, Van Reenan, Webb (2020): ideas harder to find

2. Funk, Leahey, Park (2023). it is not about researchers or inventors, but about 'a fundamental shift in the nature of science and technology'.

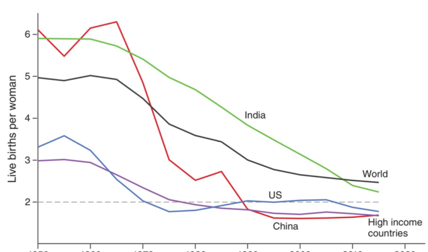


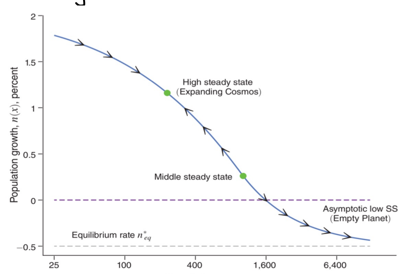


Growth and Fertility

Jones (2022) tweaks his model to introduce 2 steady states

1. Empty Planet (EP) steady state:
   declining population and stagnant living standards
2. Expanding Cosmos (EC) steady state:
   increasing population and flourishing society





Week 6:
Consumption and RBC

Standard Intertemporal Model of Consumption
From Euler Equation to IS curve
Hall Random Walk Model
- Excess sensitivity
- Income News and Permanent Income

Drawback of Certainty Equivalence
- Precautionary Saving
- Hyperbolic Discounting

RBC Model
Steady-State and Response to shocks

Intertemporal Consumption Model

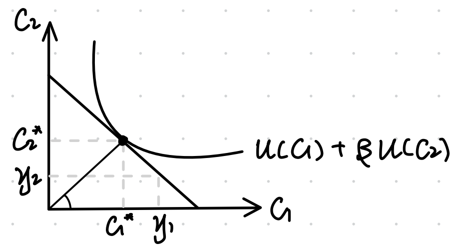
Two period consumption model:
(Household)

$\max_{\{c_1, c_2\}} u(c_1) + \beta u(c_2)$

s.t. $c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$

FOC $\Rightarrow u'(c_1) = \beta(1+r)u'(c_2)$

(or: MRS = MRT) $\frac{u'(c_1)}{\beta u'(c_2)} = 1+r$



Standard Intertemporal Model of Consumption

Utility function: $U = E\left[\sum_{t=0}^{+\infty} \beta^t u(c_t)\right]$

rational expectation → E; discount factor β; ↑ consumption occuring at time t; the curvature of the instantaneous utility function, $u(\cdot)$

1. Elasticity of Intertemporal Substitution (EIS): $EIS = -\frac{u'(c)}{cu''(c)}$
   It measures how the MRS varies with c_{t+1}/c_t (convexity of the ICs)
   The more convex the higher the desire for consumption smoothing
2. Justification for $\beta < 1$: psychological factor (e.g., impatience); chance of death
   β^t: exponential discounting, alternative assumptions include hyperbolic discounting
3. Justification for $\Sigma^{+\infty}$: 1. 'infinity' is to be taken as a convenient approximation for a finite but very long horizon
   2. Intergenerational Altruism: people care for their offspring
4. Alternative to instantaneous utility (independent of consumption in other periods):
   $u(\cdots, c_{t-1}, c_t, c_{t+1}, \cdots)$ is the more generally case.
   Thus, $\Sigma \beta^t u(c_t)$: intertemporal separability / additivity over time.
5. Weaker assumption than RE: agents form expectations in a way that ensures no systematic, i.e., predictable, mistake is made

Intertemporal Budget Constraint: $E\left[\sum_{t=0}^{+\infty} \frac{c_t}{(1+r)^t}\right] = E\left[\sum_{t=0}^{+\infty} \frac{y_t}{(1+r)^t}\right] + (1+r)A_0$

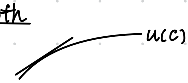
further income from working; interest rate r; household's financial position A_0 ($A_t>0$: asset, $A_t<0$: debts)

$A_{t+1} = (1+r)A_t + y_t - c_t$

$\Rightarrow \frac{A_{t+1}}{(1+r)^t} = \frac{A_t}{(1+r)^{t-1}} + \frac{y_t - c_t}{(1+r)^t} \xrightarrow{\Sigma_t}$ "no Ponzi game" condition: $\lim_{t\to+\infty} E\left[\left(\frac{1}{1+r}\right)^t A_{t+1}\right] = 0$

Euler Equation: $u'(c_t) = \beta(1+r)E[u'(c_{t+1})] \Rightarrow \frac{u'(c_t)}{E[u'(c_{t+1})]} = \beta(1+r)$

1. Euler equation: how the interest rate affects desired consumption growth
2. when $(1+r) > \frac{1}{\beta}$, MU is decreasing, so consumption is increasing
   when $(1+r) = \frac{1}{\beta}$, the household desires a flat path for consumption
3. The sensitivity of consumption growth to change in c_{t+1}/c_t associated with a given change in the MRS, i.e., it depends on the EIS.



Assume $\frac{1}{\sigma} = EIS = -\frac{u'(C)}{Cu''(C)}$, thus $u(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$ $(\sigma>0)$
Then the Euler Equation becomes $C_t^{-\sigma} = \beta(1+r)C_{t+1}^{-\sigma}$, which can be written $\frac{C_{t+1}}{C_t} = [\beta(1+r)]^{\frac{1}{\sigma}}$
- So consumption will follow an upward, flat or downward path depending on whether $\beta(1+r)$ is bigger, equal or smaller than 1.
- Sensitivity depends on $\frac{1}{\sigma}$, the EIS
- A version of this equation could also be the basis for estimating the EIS.

From the Euler Equation to the IS curve

the constant EIS utility function: $u(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$, $\sigma>0$

$\Rightarrow C_t^{-\sigma} = E[\beta(1+r_{t+1})C_{t+1}^{-\sigma}]$ (we now allow the interest rate to change over time)

$\Rightarrow -\sigma \ln C_t = \ln\beta + r_{t+1} - \sigma \ln C_{t+1}$ use approximation: $\ln(1+x) \approx x$ when x small

$\Rightarrow \ln C_t = -\frac{1}{\sigma}\ln\beta - \frac{1}{\sigma}r_{t+1} + \ln C_{t+1}$ approximation, not correct to simply reintroduce "E"

$\Rightarrow \ln C_t = -\frac{1}{\sigma}\ln\beta - \frac{1}{\sigma}E(r_{t+1}) + \ln E(C_{t+1})$ simply assume $Y=C$

$\Rightarrow \ln Y_t = -\frac{1}{\sigma}\ln\beta - \frac{1}{\sigma}E(r_{t+1}) + \ln E(Y_{t+1})$

Compare with textbook IS: $Y_t = A - ar_{t-1}$

1. The sensitivity of output growth to the interest rate is linked to the EIS
2. We can think of $-\ln\beta$ as the natural (stabilizing) real interest rate, say $\bar{r}$ / r_s

$\Rightarrow \ln Y_t = -\frac{1}{\sigma}E[r_{t+1}-\bar{r}] + \ln E[Y_{t+1}]$

$\Rightarrow \ln Y_t = -\frac{1}{\sigma}\sum_{i=1}^{T}E[r_{t+i}-\bar{r}] + \ln E[Y_{t+T}]$

Therefore, current output depends on expected future interest rate

Hall Random Walk Model

Assumption: $\beta(1+r) = 1$ (steady state)

$\Rightarrow u'(C_t) = E[u'(C_{t+1})]$, MU is expected to be constant
without uncertainty, equal MUs imply equal consumption

However in general $E[u'(C_{t+1})] \neq u'(E[C_{t+1}])$ unless linear MU (i.e., quadratic u)

Assume $u(c) = ac - \frac{b}{2}c^2$, then the Euler equation becomes:

$C_t = E(C_{t+1})$ or $C_{t+1} = C_t + \varepsilon_t$, $E(\varepsilon_t)=0$ —— Random Walk

Then $E[\sum_{t=0}^{+\infty}\frac{C_t}{(1+r)^t}] = C_0\sum_{t=0}^{+\infty}\frac{1}{(1+r)^t} = \frac{1+r}{r}C_0$

Substitute into the PV budget constraint: $\frac{1+r}{r}C_0 = E[\sum_{t=0}^{+\infty}\frac{y_t}{(1+r)^t}] + (1+r)A_0$

$\Rightarrow C_0 = rA_0 + r(\frac{1}{1+r}E[\sum_{t=0}^{+\infty}\frac{y_t}{(1+r)^t}]) = r[A_0 + H_0] \equiv y_p$ the Permanent Income Hypothesis

, where $H_0 \equiv \frac{1}{1+r}E[\sum_{t=0}^{+\infty}\frac{y_t}{(1+r)^t}]$ is human wealth
y_p is annuity value of total wealth, often referred as permanent income

1. marginal propensity to consume out of total wealth is r.
2. agents consume a flow of resources that leaves total wealth unchanged
3. Certainty Equivalence: the volatility of income doesn't affect consumption choices

Excess Sensitivity

We can estimate $\Delta C_{t+1} = \gamma X_t + u_t$ (consumption follows a random walk)

X_t: variables that might predict income changes, but already in the agents' information set

If the model is correct, $\gamma = 0$

- Hall: only equity returns significant
- Flavin (1981): consumption respond to past income changes
  — excess sensitivity of consumption to past information

  But correlation found using aggregate data may be spurious:
  if all agents keep consumption constant but mean incomes increases across generations, aggregate consumption will be perfectly correlated with income.
- Zeldes (1989): evidence of excess sensitivity for households for which credit constraints were likely to bind

Cross-sectional data is also vulnerable to spurious results. Under the permanent income hypothesis, the conditional expectation of the error term ought to be zero, Et [ut] = 0. But if changes in incomes are the results of both aggregate and idiosyncratic shocks, then there is no guarantee that the average error will be zero even in a large cross section

- Campbel (1987), Japelli & Pistaferri (2000): weak or no excess sensitivity
- Souleles (1999), Smilarly Hsieh (2003): excess sensitivity w.r.t. tax

Income News and Permanent Income

A simple model for income: $y_t = \alpha + \rho y_{t-1} + \mu_t$, $E_t(\mu_t) = 0$

model: aimed at prediction, not explanation; ρ: persistence of income changes

Since $E_t[y_t] = \alpha + \rho y_{t-1}$, if there is a shock $\mu_0 = y_0 - E_0(y_0) \neq 0$

$$\Rightarrow \begin{cases} E_0(y_1) = \alpha + \rho E_0(y_0) + E_0(\mu_1) = \alpha + \rho E_0(y_0) + \rho \mu_0 \\ E_0(y_2) = \alpha + \rho(\alpha + \rho E_0(y_0)) + E_0(\mu_2) = (1+\rho)\alpha + \rho^2 E_0(y_0) + \rho^2 \mu_0 \end{cases}$$

Therefore, $E_0[y_t | \mu_0] - E_0[y_t] = \rho^t \mu_0$

the change in expectation about future incomes triggered by μ_0 at time 0.

An income shocks μ_0 ⇒ changes future income forecasts ⇒ change our estimate of human wealth ⇒ changes permanent income

$$\Delta C_0 = \frac{r}{1+r}\Delta = \frac{r}{1+r}\sum_{t=0}^{+\infty} \frac{\mu_0 \rho^t}{(1+r)^t} = \frac{r}{1+r-\rho}\mu_0$$

— The effect of a shock on consumption is larger the more persistent income shocks are perceived to be

Campbel and Deaton (1989): aggregate data consistent with $\rho = 1$.

⇒ $\Delta C_0 = \mu_0$, consumption should be just as volatile as income

However: excess smoothness — c significantly less volatile than y.

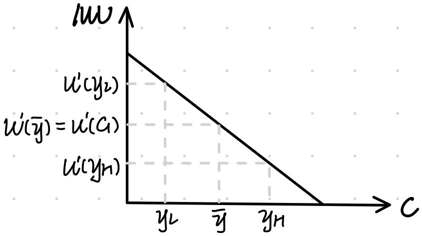
Ignorance of Certainty Equivalence

Excess sensitivity + excess smoothness: simple certainty equivalence model might neglect something important.

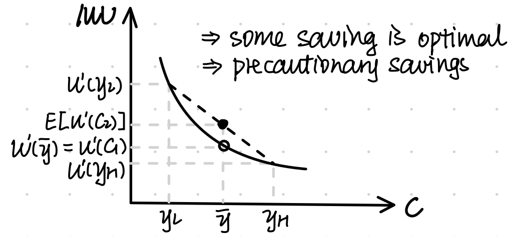
Precautionary Saving

Assume $\beta(1+r)=1$, $y_1=\bar{y}$ for sure. y_2 may be y_H or y_L with equal probability. $(y_H+y_L)/2=\bar{y}$. It's optimal to set $c_1=y_1=\bar{y}$ without uncertainty.

Quadratic utility: $u'''(c)=0$, with no saving $u'(c_1)=E[u'(c_2)]$



Non-Quadratic utility: $u'''(c)>0$, with no saving $u'(c_1)<E[u'(c_2)]$



Explain excess sensitivity:
the variance of income matter for consumption choices if variance is correlated with growth ⇒ excess sensitivity

Explain excess smoothness:
more prudent consumers responds less sharply to income shocks because use accumulated wealth to smooth shocks

Hyperbolic Discounting

Empirical Study: people have problems of lack of self control and time inconsistency

Main idea: the marginal propensity of consume out of different types of incomes and asset is different.

Model of hyperbolic discounting:

$$U=u(c_0)+\theta E\left[\sum_{t=1}^{+\infty}\beta^t u(c_t)\right],\quad \theta>1$$

utility at time 1 relative to time 0 (present) is discounted more: $\theta\beta>\beta$

⇒ time inconsistent choices

But in this model the agent is aware of these inconsistency, and attempts to limit their temptation for instant gratification. So they purchase assets that are difficult to liquidise, put their savings in plans that penalise early withdrawals, etc.

Real Business Cycles (RBC)

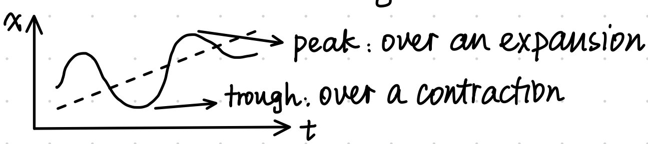
Business Cycles: Temporary and recurrent deviations of output and employment from trend

Any economic variable: $x_t = \underbrace{S_t}_{\text{smoother trend component}} + \underbrace{V_t}_{\text{a more volatile cyclical component}}$

Contraction: any period with a negative cyclical component

Terminology

1. A variable is said to be pro-cyclical if $Corr(V_t, y_t) > 0$.
2. A variable is said to be counter-cyclical if $Corr(V_t, y_t) < 0$.
3. Else the variable is a-cyclical



For the US economy, King and Rebelo (1999) report the following

		Standard deviation	Correlation with output
output	Y	1.81	1.00
consumption	C	1.35	0.88
investment	I	5.30	0.80
labour	L	1.79	0.88
GDP per labour	Y/L	1.02	0.55
wage rate	w	0.68	0.12
interest rate	r	0.30	-0.35
technology	A	0.98	0.78

All variables logged and detrended. See King and Rebelo (1999) for details.

Model Structure: impulse $\Rightarrow$ propagation mechanism $\Rightarrow$ cycles

impulse — often: supply shocks due to randomness in exogenous technological progress

propagation mechanism — emphasize the rational responses to shocks of households, firms and policy makers

Key points: the allocation of time between production and leisure, and of output between consumption and investment.

Structure

Standard neoclassical production function: $Y_t = \underbrace{A_t}_{\text{total factor productivity, subject to random shocks}} \underbrace{F(K_t, L_t)}_{\text{constant RTS}}$

Perfect factor markets and profit maximization.

$$\begin{cases} r_t = A_t \dfrac{\partial F(K_t, L_t)}{\partial K_t} \equiv MPK_t \\ W_t = A_t \dfrac{\partial F(K_t, L_t)}{\partial L_t} \equiv MPL_t \end{cases}$$

$\Rightarrow$ const RTS: $r_t K_t + W_t L_t = A_t F(K_t, L_t) = Y_t$

assume a Cobb-Douglas function: $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, $0 < \alpha < 1$

Thus: $r_t = A_t \alpha \left(\frac{K_t}{L_t}\right)^{\alpha - 1} \equiv MPK_t$

$W_t = A_t (1-\alpha) \left(\frac{K_t}{L_t}\right)^{\alpha} \equiv MPL_t$

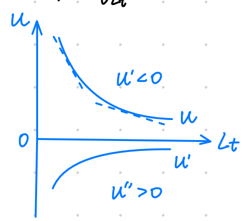
Household intertemporal utility function: $\max E\left[\sum_{t=0}^{+\infty} \beta^t u(C_t, \underbrace{L_t}_{\text{hours worked}})\right]$

hours worked, $\frac{\partial u(C_t, L_t)}{\partial L_t} < 0$, $\frac{\partial^2 u(C_t, L_t)}{\partial L_t^2} > 0$

subject to a series of capital accumulation equations:

$$K_{t+1} = r_t K_t + W_t L_t - C_t + (1-\delta) K_t$$

or: $K_{t+1} = Y_t - C_t + (1-\delta) K_t$



$\Rightarrow$ FOCs for C_t: **consumption Euler Equation**

$$\boxed{\frac{\partial u(C_t, L_t)}{\partial C_t} = \beta E\left(\frac{\partial u(C_{t+1}, L_{t+1})}{\partial C_{t+1}} (1 - \delta + r_{t+1})\right)}$$

Intuition: { LHS: current MU of consumption
RHS: marginal benefit of saving

$\Rightarrow$ FOC for labour: **intratemporal labour supply equation**

$$\boxed{W_t \frac{\partial u(C_t, L_t)}{\partial C_t} = -\frac{\partial u(C_t, L_t)}{\partial L_t}}$$

Intuition: { RHS is the loss in utility caused by working marginally harder
But this extra effort allows me to increase consumption by W_t
LHS is the benefit in terms of increased utility

$\Rightarrow$ Combine the above FOCs: **labour supply Euler Equation**

$$\boxed{-\frac{\partial u(C_t, L_t)}{\partial L_t} = -\beta E\left[\frac{\partial u(C_{t+1}, L_{t+1})}{\partial L_{t+1}} \cdot \frac{W_t}{W_{t+1}} (1 - \delta + r_{t+1})\right]}$$

Intuition: { LHS: marginally reducing work effort this period increases by the marginal disutility of work.
RHS: earn less $\Rightarrow$ save less $\Rightarrow$ lower capital stock at time $t+1$ thus harder work is needed at $t+1$.

Punchline: $\frac{W_t}{W_{t+1}} \uparrow \Rightarrow$ work harder now ($L_t \uparrow$) and enjoy more leisure later

Total Factor Productivity, A_t is subject to random shocks:

$$A_t = e^{z_t}, \text{ where } z_t = \underbrace{\rho}_{\text{measure of persistence}} z_{t-1} + \underbrace{u_t}_{\text{technology shock}}, \quad 0 \le \rho \le 1$$

Absent shocks, i.e., if $u_t = 0$ all t, z would converge to 0 and A to 1; with random shocks these variables hover around their equilibrium values.

$\Rightarrow$ Empirical counterpart: Solow residual from growth accounting

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \Rightarrow \frac{dY_t}{Y_t} = \frac{dA_t}{A_t} + \alpha \frac{dK_t}{K_t} + (1-\alpha)\frac{dL_t}{L_t}$$

$$\Rightarrow \frac{dA_t}{A_t} = \frac{dY_t}{Y_t} - \alpha \frac{dK_t}{K_t} - (1-\alpha)\frac{dL_t}{L_t}$$

"the measure of our ignorance": changes in output that we are unable to explain

1. Steady State

Consumption Euler Equation:

$$\frac{\partial u(C_t, L_t)}{\partial C_t} = \beta E\left(\frac{\partial u(C_{t+1}, L_{t+1})}{\partial C_{t+1}} (1 - \delta + r_{t+1})\right)$$

$$\Rightarrow \beta(1 - \delta + r) = 1, \quad r = \frac{1}{\beta} - 1 + \delta$$

Determine the steady-state r and therefore the steady-state K/L, w.

$\Rightarrow$ use intratemporal labour supply condition & the resource constraints to pinpoint C, L, K, Y

2. Response to a shock when $\rho = 0$

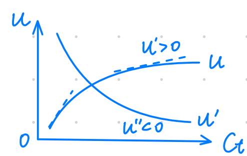
Starting from steady-state, assume a positive productivity shock in the initial period, $u_0 > 0$, so that $z_0 > 0$, $A_0 > 1$.
(TFP is temporarily higher than the steady-state value)

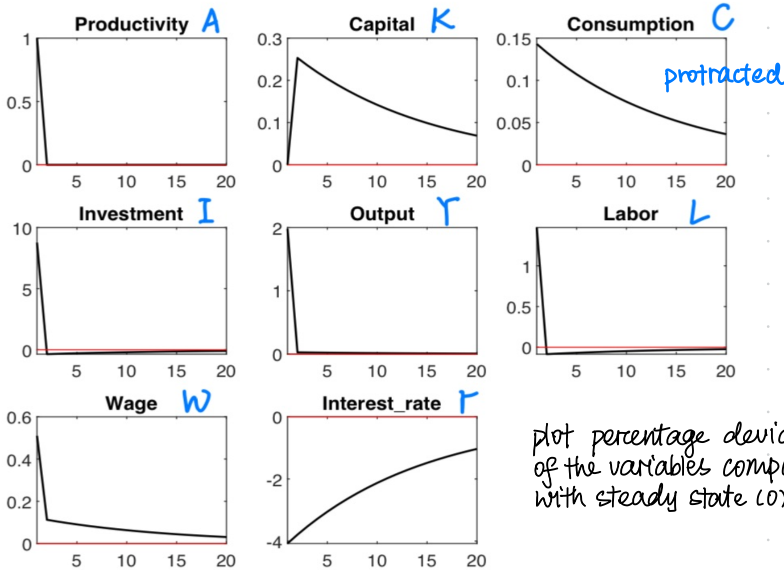
$$A_0 \uparrow \Rightarrow MPL_0 \uparrow \Rightarrow \underbrace{w_0 \uparrow \Rightarrow L_0 \uparrow}_{\text{temporarily: substitution effects > income effects}} \Rightarrow Y_0 \uparrow \Rightarrow \begin{cases} C_0 \uparrow \\ I_0 \uparrow \ (K \uparrow) \end{cases}$$

High volatility: "animal spirits" (Keynes)

$A_1 -$, $K_1 \uparrow$: expected future real interest rate $r < r_0$, $r \uparrow$
$\Rightarrow$ C decreases through time

—— capital accumulation alone does not produce substantial internal propagation of temporary productivity shocks
(there's some propagation, but it's weak)

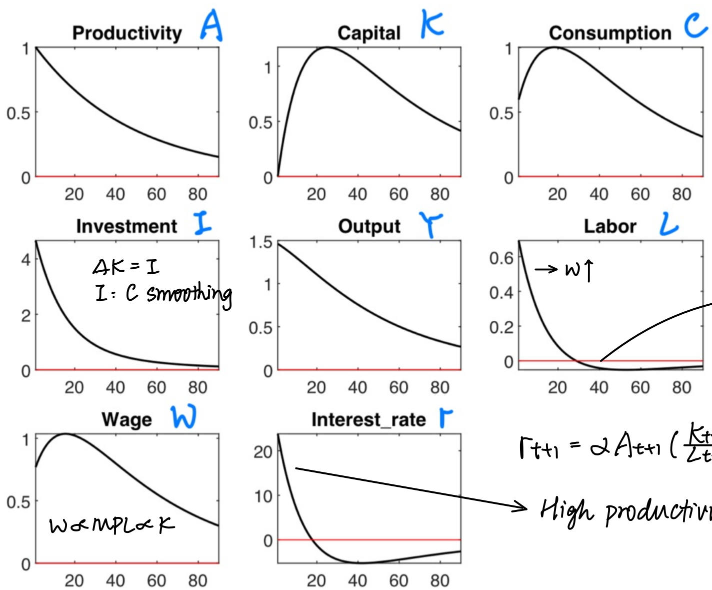




plot percentage deviations of the variables compared with steady state (0)

3. Response to a shock when $\rho > 0$

Assume that technology shock persistent, i.e., $\rho > 0$
In fact to work well, the standard RBC requires $\rho \to 1$ (e.g. $\rho = 0.979$)



correlated with r:
$r\downarrow \Rightarrow C\uparrow$

substitution effect
($w\uparrow$ to steady state)
+ $r\downarrow$ discourages
work effort

$$r_{t+1} = \alpha A_{t+1} \left(\frac{K_{t+1}}{L_{t+1}}\right)^{\alpha - 1}$$

High productivity

Empirical Evidence

	US data		Model	
	Standard deviation	Correlation with output	Standard deviation	Correlation with output
Y	1.81	1.00	1.39	1.00
C	1.35	0.88	0.61	0.94
I	5.30	0.80	4.09	0.99
L	1.79	0.88	0.67	0.97
Y/L	1.02	0.55	0.75	0.98
w	0.68	0.12	0.75	0.98
r	0.30	-0.35	0.71	0.95
A	0.98	0.78	0.72	1.00

All variables logged and detrended. See King and Rebelo (1999) for details.

fairly close except for w, r

→ contracts ⇒ wage smoothing

Facts: real wages at best very mildly pro-cyclical
labour supply strongly pro-cyclical
with the assumption of a perfectly competitive labour market,
⇒ requires a very elastic labour supply

Micro-macro paradox:
Labour supply elasticity implied by the baseline RBC model is much more higher than that usually estimated at micro level

Solutions: household labour supply decision is mostly between employment or non-employment, rather than hours worked by each.

Extensions to RBC:

1. 3-equation NKM (Clarida et al.): RBC + nominal rigidities (e.g. Calvo pricing) minus capital accumulation
2. Dynamic Stochastic General Equilibrium (DSGE): addition of various features

AD shocks

- RBC models emphasise technology shocks
- Aggregate demand shocks could be introduced, but generally are found not to give plausible results in this framework
- For example shocks to government spending cannot, by themselves, produce realistic patterns of comovement among macroeconomic variables. This result stems from the fact that an increase in government expenditures (financed with lump sum taxes) gives rise to a negative wealth effect that induces consumption to fall at the same time that labor and output rise. Thus, if government spending were the only shock in the model, consumption would be countercyclical, contrary to the data
- Changes in labour and capital income taxes have effects that are similar to productivity shocks. However, these taxes change infrequently making them poor candidates for sources of business cycles fluctuations
- It is possible to introduce money in a RBC model, e.g. via a cash-in-advance constraint (agents need money to carry out transactions); but quantitatively monetary policy shocks have small effects in this class of models
- Hence the emphasis on supply shocks

Week 7:
Fiscal Policy and Debt

Fiscal Policy as stabilization tool
- traditional Keynesian positions
- Ricardian Equivalence

Sovereign debt
- normative perspectives on sovereign debt management
- austerity measures to reduce national debt
- upward trends in national indebtness

Fiscal Policy (Keynesian & Ricardian)

Keynesian view:

Budget Deficits ($G\uparrow$ or $T\downarrow$) $\Rightarrow$ shift out IS curve

- sticky prices $\Rightarrow$ output / employment rise in short-run
- trade deficit and increased international borrowing follow
  $\Rightarrow$ real exchange rate appreciates

Ricardian view.

tax cuts financed from borrowing do not shift IS

rises in government purchases shift IS but multiplier ≤ 1

1. the Effect of a tax cut on the Current Account

$\Delta T_1 < 0,\ \Delta G_1 = \Delta G_2 = 0,\ B_0^g = 0$

$$G_1 + \frac{G_2}{1+r_1} = T_1 + \frac{T_2}{1+r_1} + (1+r_0)B_0^g$$

$$\Rightarrow \Delta G_1 + \frac{\Delta G_2}{1+r_1} = \Delta T_1 + \frac{\Delta T_2}{1+r_1}$$

$$\Rightarrow \Delta T_2 = -(1+r^*)\Delta T_1 > 0$$

Thus, $\Delta C_1 = 0,\ \Delta S_1^P = -\Delta T_1 > 0$

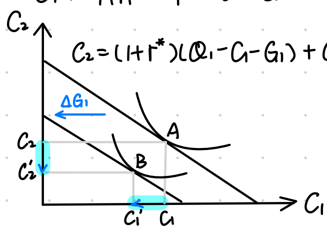
National Saving $= S_1 = S_1^P + S_1^P \Rightarrow \Delta S_1 = 0$. $CA_1 = S_1$, unchanged

—— When Ricardian Equivalence holds and the final deficit is the result of a tax cut, then the Twin Deficit Hypothesis fails.

2. A Temporary Increase in Government Spending

$G_1\uparrow$ and $\Delta G_2 = 0,\ B_0^* = 0$

$$C_1 + \frac{C_2}{1+r_1} = \tilde{Y} = Q_1 - G_1 + \frac{Q_2 - G_2}{1+r_1} + (1+r_0)B_0^*$$



$\Rightarrow 0 > \Delta C_1 > \Delta G_1$

$TB_1 = Q_1 - C_1 - G_1$ deteriorates, but $\Delta TB_1 > -\Delta G_1$

$CA_1 = TB_1 + r_0 B_0^*$ deteriorates

—— Twin Deficit Hypothesis

Ricardian Equivalence

Assumptions:

1. Government budget constraints: $\sum_{t=1}^{+\infty} \frac{G_t}{(1+r)^{t-1}} = \sum_{t=1}^{+\infty} \frac{T_t}{(1+r)^{t-1}}$

   G: real expenditure, T: real lump-sum taxation, r: real interest rate

2. Fixed population of infinitely-lived, homogeneous, risk neutral agents maximizing lifetime utility.
   (perfect consumption smoothing, for simplicity)

3. zero substitutability of c, G (to be relaxed)
4. perfect capital markets: consumers, government face same r, r exogenous to domestic saving decisions, e.g., world rate

Debt financed tax cuts

First period tax cut: $T_1 < \bar{G}$, borrowing $\bar{G} - T_1$

Solvency requires tax rise in period n: $(\bar{G} - T_1)(1+r)^n$

$\Rightarrow$ consumption decisions unchanged, IS curve not shift

Implication: 1. Government bonds not part of net wealth
since there is a matching liability in the form of future tax liabilities (Barro, 1974)

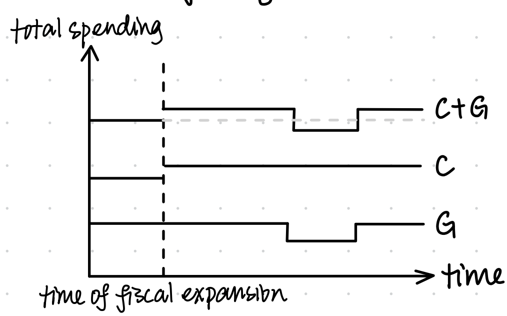
2. Private saving invested in government bonds, to finance government deficit

3. Equivalence of funding methods
irrespective of whether G is financed from taxation / debt

Tax cut financed from future government spending cut

permanent income rises $\Rightarrow$ consumption rises in all periods

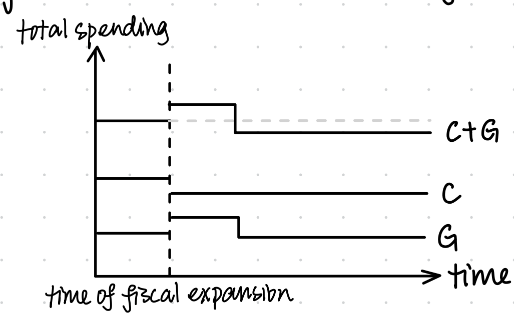
result comes from government choosing not to smooth its consumption



Caveat:
If G, C perfect substitutes, consumers will increase C one-for-one with decrease in G, and must save original tax cut in full to do this.
— tax cut then neutral as in debt finance case

Temporary G↑ financed from debt

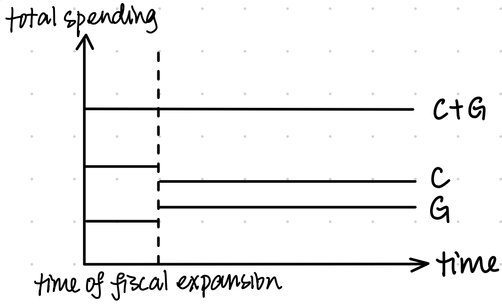
future tax increases $\Rightarrow$ saving ↑ in all periods to pay tax bill



1. IS shifts out on net today, but the multiplier defining horizontal shift of IS will be less than 1 (some G offset by private saving)
2. Budget Deficit from G↑ has different effects to budget deficit from tax cuts

Permanent G↑ financed from debt

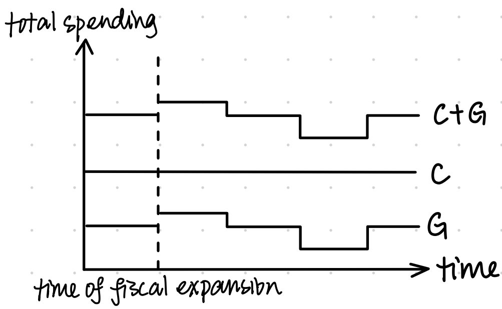
future tax increases ⇒ saving ↑ in all periods to pay tax bill



1. Total demand and the IS curve will not change in any period
2. Per period saving rise to finance fiscal expansion must be smoothed across periods from time of expansion

Temporary G↑ financed via future G↓

right shift IS today, left shift in periods afterward



1. No change in private consumption (saving), so current period spending multiplier is 1 in the example
2. Future left shift of IS larger than initial right shift due to need to pay interest on debt.

Reflections on Ricardian Equivalence

1. Tax cuts only an effective tool for macro management if backed up by future spending cuts
2. Spending increases may stimulate the economy, but also require future policy reversals.

Ricardian Equivalence may not hold in practice

1. Capital Market Imperfections

   Asymmetric information ⇒ $\begin{cases} \text{credit rationing} \\ r^c > r^g \text{ (gov. bond)} \end{cases}$

   Households: limited access to intertemporal finance methods
2. Income Uncertainty, Risk Aversion ⇒ precautionary saving

3. Finit Horizons

Barro: agents behave dynastically, maximizing PDV of their own utility and that of their future generations.
⇒ then horizons infinit even if lifetimes are not

Empirical evidence: households behave dynastically?

Bequests (遗产) often residual wealth at end of lifecycle, not by altruism
Bernheim (1989): awkward implications from dynastic behaviour.
—— transfers within private sector used to offset all changes in wealth (unrealistic)

4. Limits to individual rationality

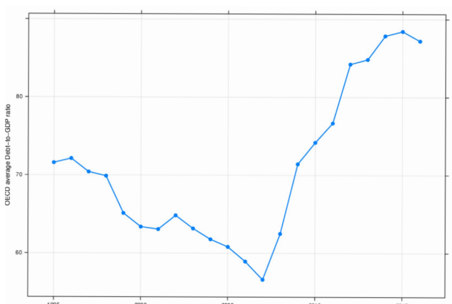
5. Heterogeneous households

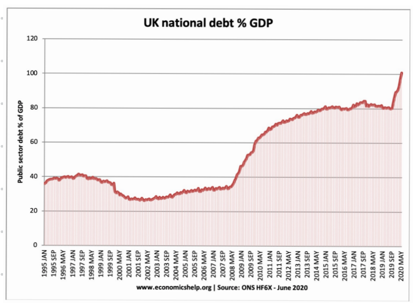
a tax cut may transfer resources to agents with higher propensity to consume from wealth ⇒ raise overall consumption

6. Distortionary Taxation

{ cuts in income tax ⇒ more labour supply today
{ cuts in sales taxes ⇒ consume harder today

Debt Sustainability





Dynamics of the debt ratio: $\Delta b = \underbrace{d}_{\text{primary budget deficit}} + (r - \gamma_y) b$, $b = \frac{\text{debt}}{\text{GDP}}$

primary budget deficit (before debt interest payment minus tax receipts)

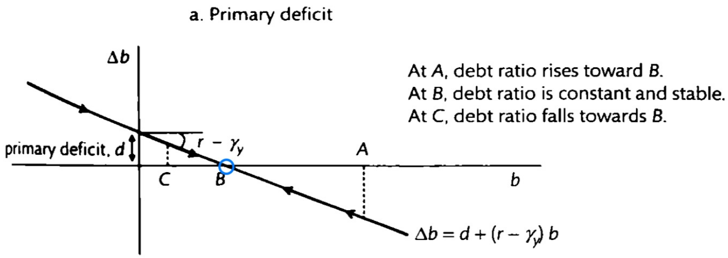
r: constant real interest rate on debt

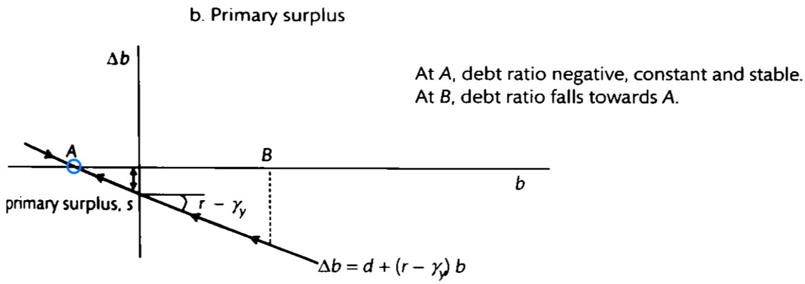
γ_y: constant growth rate of real GDP

sustainable if $(r - \gamma_y) < 0$

intuition: although there's interest to service on the debt, this accrues less quickly than GDP grows so that the debt ratio stabilizes

Caveat: shocks to $(r - \gamma_y)$ such as slower growth or higher interest rates could rapidly change the situation ⇒ risk of insolvency exists as long as $b > 0$.





if $(r-\gamma_y)>0$: can only be maintained if there is a sufficiently large primary surplus d.
—— eventually leads to unsustainable debt/GDP.
if $(r-\gamma_y)=0$: only $d=0$ is consistent with debt sustainability.

Debt/GDP ratio management

Assumptions:
1. perfect capital market (fixed r)
2. infinite government planning horizon
3. social preference is for a smooth flow of government expenditure on public goods and other services over time.
4. welfare losses from distortionary government taxation are convex in tax rates ⇒ smooth tax rates

Debt Smoothing: Optimal Choice Ex Ante

maintain the inherited debt/GDP ratio, and to neither pay down nor accumulate debt as a fraction of GDP.
(smoothing b over time should be the intention)

Ex Post: there may be shocks that necessitate more debt
(e.g. a pandemic that cuts income and increases need for vaccine research)
⇒ debt ratio ↑ to deal with the consequences
⇒ debt ratio smoothed around the new higher level

(*) Empirical: unexpected shocks of this kinds may have contributed to the upward trend in public debt across the OECD.

When should countries reduce public indebtedness?

1. **Why: Interest rate risk premium**
   (global investors charge an interest rate risk premium)
   - According to $\Delta b = d + (r - \gamma_b)b$, countries with largest b will need to run the largest primary surpluses in order to avoid an explosive debt path (shocks force $(r-\gamma_y)$ positive) on which debt rises without limit.
   - Limited primary surpluses ⇒ highly indebted nations face ever increasing debt ⇒ risk of insolvency (this could lead to self-fulfilling debt crises)

   How: gradual path to global average

   No simple answer.

   No simple link between debt ratios, interest rate premium charged and the strength of the case for debt reduction, a lot depends on:
   1. debt ownership.
      e.g., Japan: domestic creditors ⇒ not face an interest rate risk premium
   2. debt maturity: longer term ⇒ more time to raise required primary surpluses
   3. factors such as the strength of political consensus around debt reduction and the efficiency of tax collection services

2. **Why: other reasons**
   - **unfair to leave debt as a burden on future generations**
     But if debt is all domestically held, the next generation will likely inherit the corresponding assets.
     If debt ratio rises due to shocks, it's fair to spread the costs across generations
   - **inflation risk**
     $\pi \uparrow > E(\pi)$ ⇒ impaired real return, as bonds pay fixed nominal value
     High inflation of the 1970s: help clear a large chunk of the post WWII debt spike, but government bonds became known as 'certificates of confiscation' (没收)
     Also: CB compromises on higher inflation target to help government getting rid of excess debt.
   - **crowding out**
     Government bonds absorb a large fraction of the global savings, and limit the supply of funding for private sector projects.

     some go further and contend that ample supply of government bonds is crucial in meeting the needs of the financial system for safe assets, e.g. banks often invest saver deposits in government bonds and use those bonds as collateral in sourcing funding in international money markets

How are rising debt ratios to be explained?

1. In some accounts, rising public debt ⇒ optimal response to economic development
2. At ZLB. fiscal expansion supports economic recovery
3. Many forms of investment generate positive externalities.
4. Driven by political economy effects (deficit bias)

   Strategic Debt Accumulation:

   Political parties in government can limit the future actions of political opponents who replace them in o¢ ce through increasing debt and hence future repayments so that budgets do not o§er space for expenditure on political priorities

   - particularly applies in polarized political systems and those leading to frequent changes of government

   Excess output targets: political incentives to expand output ahead of elections could lead to fiscal deficits and rising debt ratios

(*) Fiscal Councils: watchdogs against inefficient debt accumulation
e.g. Office for Budget Responsibility (OBR) in the UK
scrutinize fiscal plans and expose hidden deficit bias.